"Propagation of surface Love waves in a lossy layered planar waveguide with a viscoelastic guiding layer"

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27 czerwca 2016 r., Warszawa, Polska.

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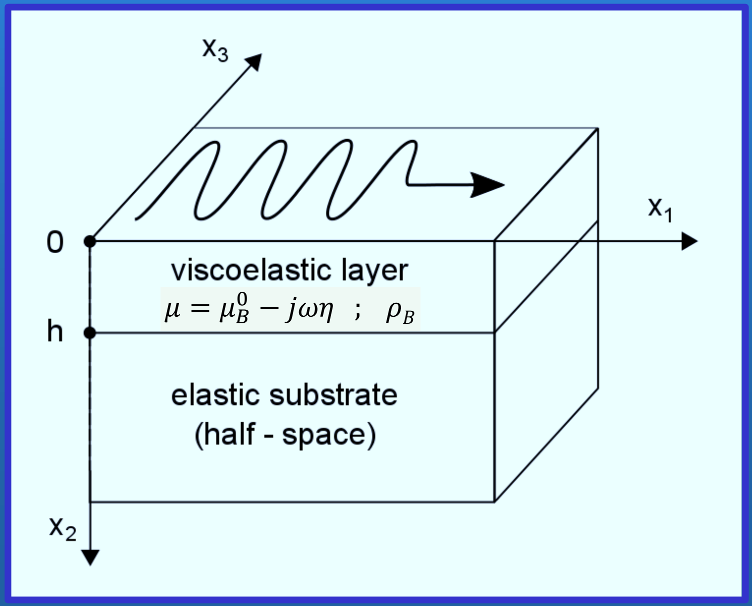
Objectives of the study

A. Scientific objective of the Study is to develop theoretical foundations and creating a mathematical model of the phenomenon of propagation of transverse surface Love waves in a layered viscoelastic media

B. In future, establishing on this basis a new non-destructive method for the identification of rheological parameters (elasticity, viscosity, density) of viscoelastic media. New method that uses surface Love waves will be non-destructive, rapid, accurate and computerized without drawbacks of classical mechanical methods.



This problem has not been solved yet in the worldwide literature

Fig.1. Geometry of the surface Love wave layered waveguide. Viscoelastic layer over an elastic substrate.

Importance of the problem

A. Large quantity of processed plastics and polymers (milion tons a year).
   Trial and Error method is still often used in the plastic industry.

B. Theory of sensors (bio and chemosensors).
   Presented in this study model can serve as a mathematical model of sensors.
   <u>To date, there is no mathematical model of sensors based on the SH waves</u>.

C. In geophysics and seismology. Exploration of natural resources. Love waves propagating in layered geological structures coverd with a liquid (e.g., Ocean).
   <u>Lack of the theory</u>.

D. In microelectronics (MEMS – Micro Electro Mechanical Systems).
   Examination of the quality of thin layers.

Effects of the Love wave action during earthquakes

Love waves are very dangerous for buildings. Shear of foundations

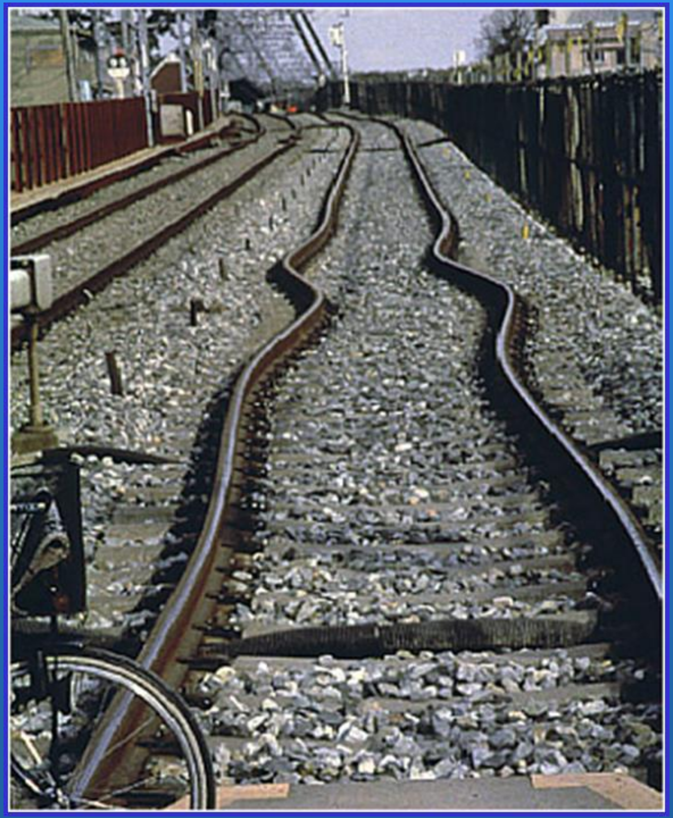


Fig.2. Illustration of Love wave activity.

Mechanical methods for determining the rheological properties of viscoelastic media

A. Couette (rotating cylinders) – 1890
B. Falling ball (Hoppler - 1932)
C. Falling sinker (e.g., needle, cone, cylinder)
D. Cone-plate
E. Capillary tube viscometer (Poiseuille)

Disadvantages:

a) Presence of moving parts
b) Require special sophisticated equipment
c) Measurements are tedious and time consuming
d) Large dimensions
e) Difficult to computerize
f) Cannot operate in real-time
g) Only laboratory methods (cannot be employed on-line)

Application of bulk ultrasonic waves for the determination of the rheological parameters of viscoelastic media

For example: Plate SH (Shear Horizontal) ultrasonic waves, Torsional waves, Lamb waves.

1. Standing waves (resonators)
   e.g., torsionally oscillating piezoelectric quartz rod, vibrating fork, vibrating cantilever (2008) – complicated setup, optical readoud

2. Travelling waves (waveguides)

   The acoustic energy is distributed in the entire volume of resonator or waveguide. The contact with an investigated viscoelastic liquid takes place on their surface.

3. Low sensitivity of sensors that use bulk ultrasonic waves.

Application of surface ultrasonic waves (i.e., Love and B-G waves) for the determination of the rheological parameters of viscoelastic media

- Love waves (1911), Bleustein-Gulyaev (B-G) waves (1968)

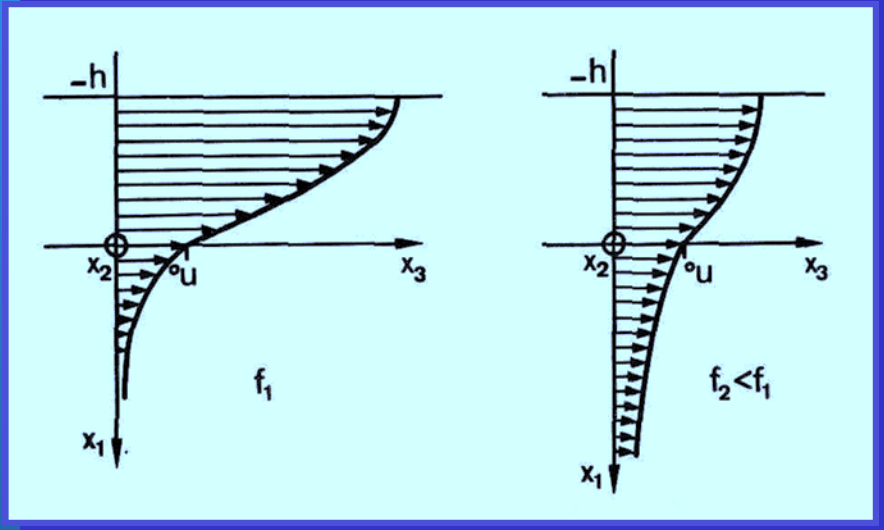
- The energy of SH-SAW is concentrated in the vicinity of the waveguide surface. Thus the SH-SAW velocity and attenuation strongly depend on the boundary conditions on the waveguide surface.

- In consequence, the sensitivity of the viscosity sensors using SH-SAW (e.g., Love waves) can be several orders larger than the sensitivity of the sensors employing bulk shear acoustic waves.

Properties of Love waves

- One component of the mechanical displacement. (Waves undergo dispersion). (Frequency range from 0.001 Hz to 500 MHz)

- Transverse (shear horizontal) surface wave
  does not exist in a homogeneous elastic half-space



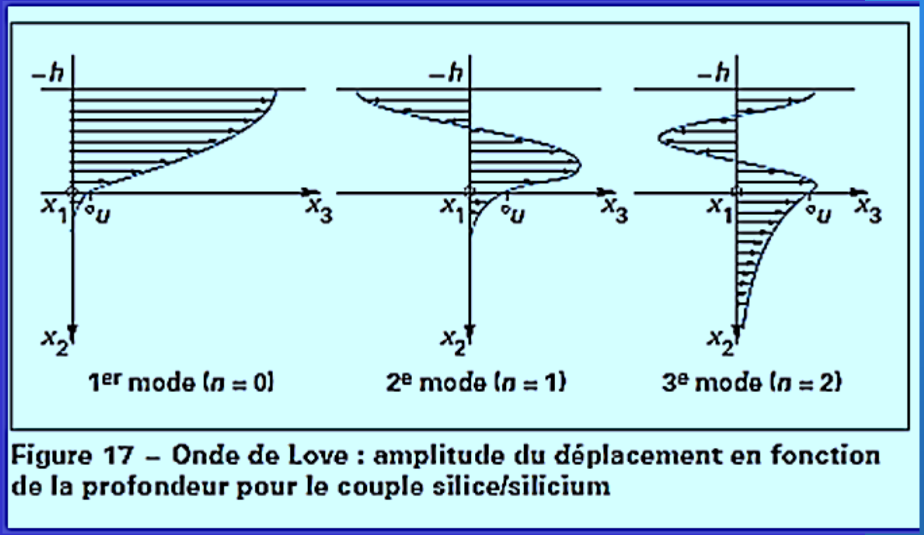


Fig.3. Distribution of the mechanical displacement with depth. $f_1 > f_2$.

Fig.4. Distribution of the mechanical displacement with depth for subsequent modes.

Advantages of Love waves sensors

1. Absence of moving parts
2. Operation in real time
3. Short measuring time
4. High sensitivity
5. Can operate at high-pressure (up to 1 GPa), and elevated temperatures (up to 400 ºC)
6. Low power consumption
7. Small dimensions, simple and robust construction of the sensor
8. Possibility of computerization
9. Output signal is electrical

Theory of Love waves propagating in viscoelastic layered media. (State of the art)

The problem of Love wave propagation in viscoelastic media is still not solved.

1) K. Sezawa, K. Kanai, Damping of periodic visco-elastic waves with increase in focal distance, Bulletin of the Earthquake Research Institute (Tokyo), 16 (1938) 491-503.

2) T. K. Das, P. R. Sengupta, L. Debnath, Effect of gravity on viscoelastic surface waves in solids involving time rate of strain and stress of higher order, International Journal of Mathematics and Mathematical Sciences, 18 (1995) 71-76.

3) S. Bhattacharaya, S.N. De, Surface waves in viscoelastic media under the influence of gravity, Australian Journal of Physics, 30 (1977) 347-354.

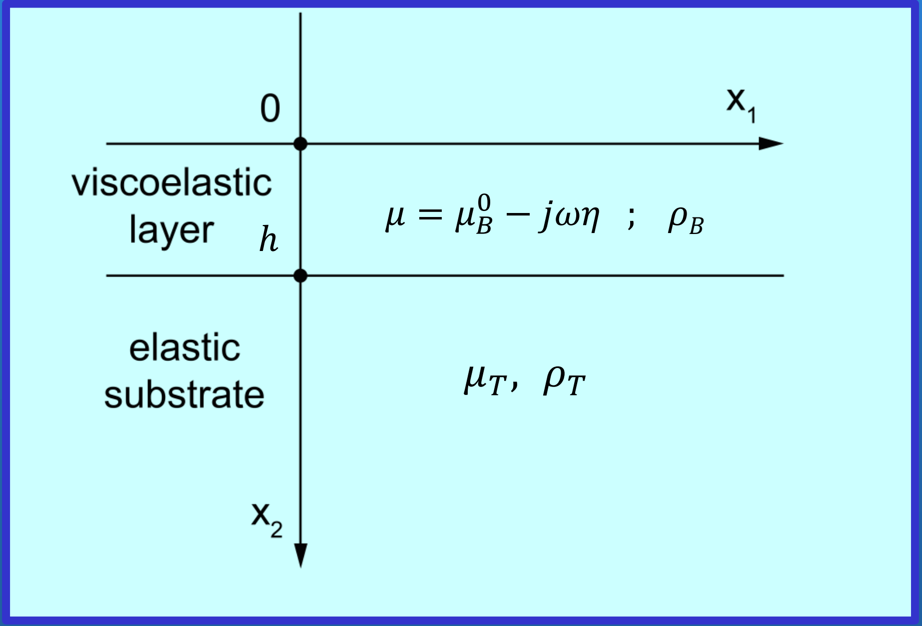
4) G. McHale, M.I. Newton, and F. Martin, Theoretical mass, liquid, and polymer sensitivity of acoustic wave sensors with viscoelastic guiding layers, Journal of Applied Physics, 93 (2003) 675-690.

Mathematical Model: Direct Sturm-Liouville Problem

Direct Sturm-Liouville Problem for Love's wave propagating in the layered viscoelastic waveguide consists in determining the phase velocity and attenuation of Love wave, knowing all the material parameters of the waveguide for a fixed frequency.

$\mu = \mu_B^0 - j\omega\eta$ - complex shear modulus; $tan\delta = \omega\eta/\mu_B^0$ - loss tangent;
δ is a phase shift between stress and strain; $j = (-1)^{1/2}$.

$k = k_0 + j\alpha$ - complex wave numer of the Love wave



Material parameters

Wave velocity and attenuation

Fig.5. Love wave waveguide with a viscoelastic surface layer.

Assumptions

1. We consider (fundamental) first mode (a kind of vibration) of Love waves
2. The substrate is an elastic, isotropic, homogeneous and lossless medium
3. The surface layer is a viscoelastic medium
4. There is no variation along the axis (x_3)
5. Losses are introduces only by the presence of a viscoelastic medium

Mathematical model: Differential equations of motion

In viscoelastic surface layer $(h > x_2 > 0)$:

$$\boxed{\frac{1}{v_1^2}\frac{\partial^2 u_3}{\partial t^2} = \left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}\right)u_3} \qquad (1)$$

where: $v_1 = ((\mu_B^0 - j\omega\eta_{44})/\rho_B)^{1/2} = v_1^0\left(1 - j\frac{\omega\eta_{44}}{\mu_B^0}\right)^{1/2}$ is the complex bulk shear wave velocity in the layer

In elastic substrate $(x_2 > h)$:

$$\boxed{\frac{1}{v_2^2}\frac{\partial^2 u_3}{\partial t^2} = \left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}\right)u_3} \qquad (2)$$

$v_2 = (\mu_T/\rho_T)^{1/2}$ is the bulk shear wave velocity in the substrate.

Propagation wave solution

In viscoelastic surface layer: $(h > x_2 > 0)$

Mechanical displacement:

$$u_3^{(1)} = W(x_2) \cdot exp[j(k \cdot x_1 - \omega t)] \quad (3)$$

$$W''(x_2) - (k_1^2 - k_0^2) \cdot W(x_2) = 0 \quad (4)$$

We postulate the solution in the form q_B:

$$W(x_2) = C_1 \cdot \sin(q_B \cdot x_2) + C_2 \cdot \cos(q_B \cdot x_2) \quad (5)$$

where: $q_B = (k_1^2 - k^2)^{1/2}$ $\quad k_1 = \frac{\omega}{v_1}$; $\quad C_1$ and C_2 are arbitrary constants

Shear stress component:

$$\tau_{23}^{(1)} = \mu_B \frac{\partial u_3^{(1)}}{\partial x_2} = [C_1 \cdot \mu_B \cdot q_B \cdot \cos(q_B \cdot x_2) - C_2 \cdot \mu_B . q_B \cdot \sin(q_B \cdot x_2)] \cdot exp[j(kx_1 - \omega t)]$$

(6)

In elastic substrate: $(x_2 > h)$

Mechanical displacement:

$$u_3^{(2)} = U(x_2) \cdot exp[j(k \cdot x_1 - \omega t)] \qquad (7)$$

$$U''(x_2) - (k^2 - k_2^2) \cdot U(x_2) = 0 \qquad (8)$$

$$U(x_2) = C_3 \cdot exp(-b \cdot x_2) \qquad (9)$$

where: $b = (k^2 - k_2^2)^{1/2}$ $\quad k_2 = \frac{\omega}{v_2}$ $\quad Re(b) > 0$

C_3 is an arbitrary constant

Shear stress component:

$$\tau_{23}^{(2)} = \mu_T \frac{\partial u_3^{(2)}}{\partial x_2} = C_3 \mu_T (-b) \cdot exp(-b \cdot x_2) \cdot exp[j(kx_1 - \omega t)] \qquad (10)$$

Boundary conditions

1. On a free surface ($x_2 = 0$), the transverse shear stress $\tau_{23}^{(1)}$ is equal to zero, hence:

$$\left.\tau_{23}^{(1)}\right|_{x_2=0} = 0 \tag{11}$$

2. Continuity of the displacement field u_3 and stress τ_{23} at the interface between the viscoelastic layer and the substrate ($x_2 = h$):

$$\left.u_3^{(1)}\right|_{x_2=h} = \left.u_3^{(2)}\right|_{x_2=h} \tag{12}$$

$$\left.\tau_{23}^{(1)}\right|_{x_2=h} = \left.\tau_{23}^{(2)}\right|_{x_2=h} \tag{13}$$

3. $u_3 = 0$ *when* $x_2 \to \infty$.

Complex dispersion equation

After substitution of Eqs. (5), (9) and (6), (10) into boundary conditions (11-13), the set of three linear and homogeneous equations for unknown coefficients C_1, C_2, and C_3 is obtained.

[M] = 3x3 Matrix

$$[M] \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = 0 \qquad (14)$$

Necessary condition for nontrivial solution is that the determinant of this matrix M equals zero.

$$\det [M] = 0 \qquad (15)$$

This leads to the following complex dispersion relation:

Complex dispersion equation
Analytical form

$$\boxed{\sin(q_B \cdot h) \cdot \mu_B \cdot q_B - \cos(q_B \cdot h) \cdot \mu_T \cdot b = 0} \qquad (16)$$

In Eq.16 the quantities $\mu_B, b \ and \ q_B$ are complex:

$$q_B = \sqrt{\left(K_1^2 \frac{1}{(1+tan^2\delta)} - k_0^2 + \alpha^2\right) + j \cdot \left(K_1^2 \frac{tan\delta}{(1+tan^2\delta)} - 2 \cdot k_0 \cdot \alpha\right)} \qquad (17)$$

$$b = \sqrt{(k_0^2 - \alpha^2 - k_2^2) + j \cdot 2 \cdot k_0 \cdot \alpha} \qquad (18)$$

$$\mu_B = \mu_B^0 - j\omega\eta_{44} = \mu_B^0(1 - j tan\delta) \qquad (19)$$

where: $K_1 = \omega / v_1^0$; $k_2 = \frac{\omega}{v_2}$; $k_0 = \frac{\omega}{v_p}$ and $tan\delta = \left(\frac{\omega\eta_{44}}{\mu_B^0}\right)$ are real variables.

Separating real and imaginary parts of the complex dispersion equation (16), we obtain:

Real part:

$$\left(\sqrt{\left(K_1^2\frac{1}{(1+tan^2\delta)}-k_0^2+\alpha^2\right)}+\frac{\left(\frac{1}{2}K_1^2\frac{tan\delta}{(1+tan^2\delta)}-k_0\cdot\alpha\right)\cdot tan\delta}{\sqrt{\left(K_1^2\frac{1}{(1+tan^2\delta)}-k_0^2+\alpha^2\right)}}\right)-\cot(c\cdot h)\cdot\tanh(d\cdot h)$$

$$\cdot\left(\frac{\left(\frac{1}{2}K_1^2\frac{tan\delta}{(1+tan^2\delta)}-k_0\cdot\alpha\right)}{\sqrt{\left(K_1^2\frac{1}{(1+tan^2\delta)}-k_0^2+\alpha^2\right)}}-\sqrt{\left(K_1^2\frac{1}{(1+tan^2\delta)}-k_0^2+\alpha^2\right)}\cdot tan\delta\right)-\frac{\mu_T}{\mu_B^0}$$

$$\cdot\left(\cot(c\cdot h)\cdot\sqrt{(k_0^2-\alpha^2-k_2^2)}+\frac{k_0\cdot\alpha}{\sqrt{(k_0^2-\alpha^2-k_2^2)}}\cdot\tanh(d\cdot h)\right)=0 \quad (20)$$

Imaginary part:

$$\cot(c\cdot h)\cdot\tanh(d\cdot h)\cdot\left(\sqrt{\left(K_1^2\frac{1}{(1+tan^2\delta)}-k_0^2+\alpha^2\right)}+\frac{\left(\frac{1}{2}K_1^2\frac{tan\delta}{(1+tan^2\delta)}-k_0\cdot\alpha\right)\cdot tan\delta}{\sqrt{\left(K_1^2\frac{1}{(1+tan^2\delta)}-k_0^2+\alpha^2\right)}}\right)$$

$$+\left(\frac{\left(\frac{1}{2}K_1^2\frac{tan\delta}{(1+tan^2\delta)}-k_0\cdot\alpha\right)}{\sqrt{\left(K_1^2\frac{1}{(1+tan^2\delta)}-k_0^2+\alpha^2\right)}}-\sqrt{\left(K_1^2\frac{1}{(1+tan^2\delta)}-k_0^2+\alpha^2\right)}\cdot tan\delta\right)-\frac{\mu_T}{\mu_B^0}$$

$$\cdot\left(\cot(c\cdot h)\cdot\frac{k_0\cdot\alpha}{\sqrt{(k_0^2-\alpha^2-k_2^2)}}-\sqrt{(k_0^2-\alpha^2-k_2^2)}\cdot\tanh(d\cdot h)\right)=0 \tag{21}$$

where:

$$c=\sqrt{\left(K_1^2\frac{1}{(1+tan^2\delta)}-k_0^2+\alpha^2\right)}\quad,\quad d=\frac{\left(\frac{1}{2}K_1^2\frac{tan\delta}{(1+tan^2\delta)}-k_0\cdot\alpha\right)}{\sqrt{\left(K_1^2\frac{1}{(1+tan^2\delta)}-k_0^2+\alpha^2\right)}}$$

Separating real and imaginary parts of the complex dispersion equation (18), we obtain:

$$\boxed{A(\mu_B^0,\ \rho_B, \mu_T,\ \rho_T, \eta_{44},\ h,\ \omega;\ k_0, \alpha\ ) = 0} \qquad (22)$$

$$\boxed{B(\mu_B^0,\ \rho_B, \mu_T,\ \rho_T, \eta_{44},\ h,\ \omega;\ k_0, \alpha\ ) = 0} \qquad (23)$$

This is a system (22-23) of two nonlinear algebraic equations.
The unknowns are: (k_0 and α).
The parameters are: $\mu_B^0,\ \rho_B, \mu_T, \rho_T,\ \eta_{44},\ h\ and\ \omega$.

$k = k_0 + j\alpha$ - complex wave numer of the Love wave

$v_p = \omega/k_0$ - Love wave phase velocity

α - Love wave attenuation in Np/m

Modified Powell hybrid method: Program MATHCAD and SCILAB

Numerical calculations

Material parameters:

For (Polymethylmethacrylate) PMMA

$\mu_B^0 = 1.43 \cdot 10^9 \; N/m^2$

$\rho_B = 1.18 \cdot 10^3 \; kg/m^3$

$v_1 = (\mu_B^0/\rho_B)^{1/2} = 1100 \; m/s$

For Quartz

$\mu_T = 5.4 \cdot 10^{10} \; N/m^2$

$\rho_T = 2.2 \cdot 10^3 \; kg/m^3$

$v_2 = (\mu_T/\rho_T)^{1/2} = 4954 \; m/s$

Thickness h = from 20 μm to 1200 μm

Program Mathcad and Scilab

Numerical calculations were performed in the range:

1) f = from 1 to 5 MHz
2) η = from 0.1 to 50 Pa s
3) $tan\delta$ = from 0 to 0.8

Love wave phase velocity dispersion curves

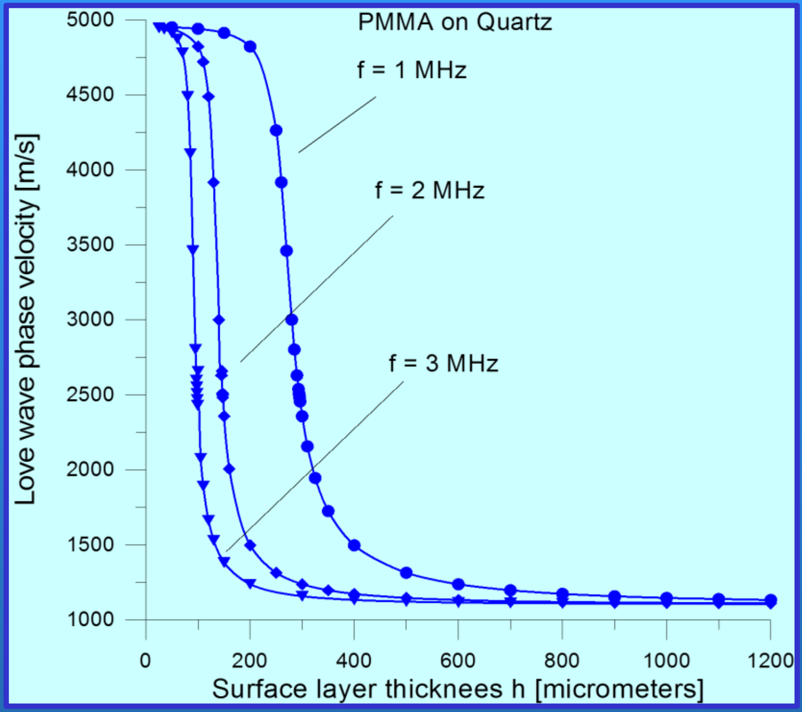


Fig.6. Phase v_p velocity versus surface layer thickness h, $\eta_{44} = 37 \cdot 10^{-2}\ Pas$.

Love wave phase velocity dispersion curves

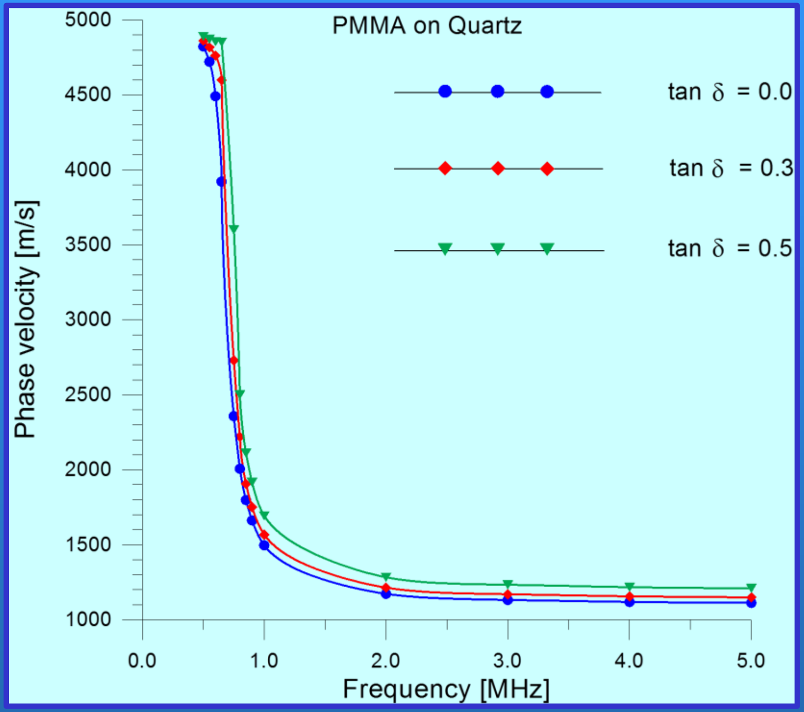


Fig.7. Phase velocity v_p versus frequency, $h = 400\ \mu m$.

Love wave phase velocity dispersion curves

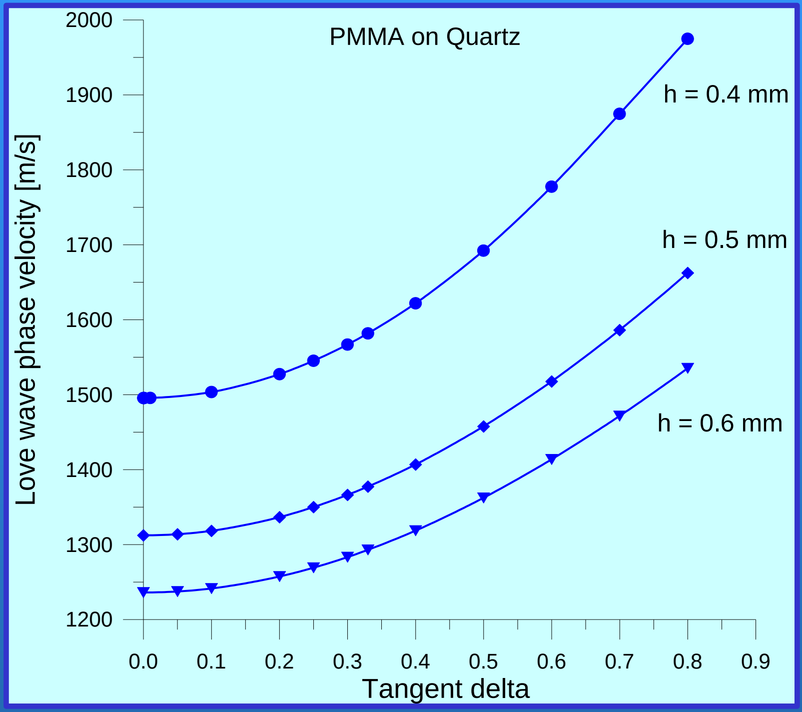


Fig.8. Phase velocity v_p versus loss tangent of the surface layer, $f = 1\ MHz$.

Love wave phase attenuation dispersion curves

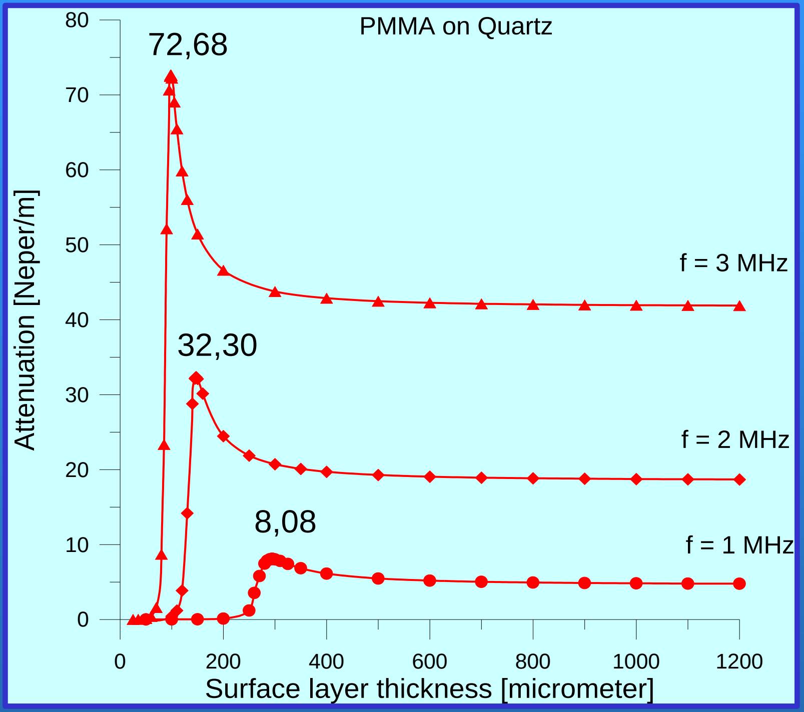


Fig.9. Attenuation α versus surface layer thickness h, $\eta_{44} = 37 \cdot 10^{-2}\ Pas$.

Love wave phase attenuation dispersion curves

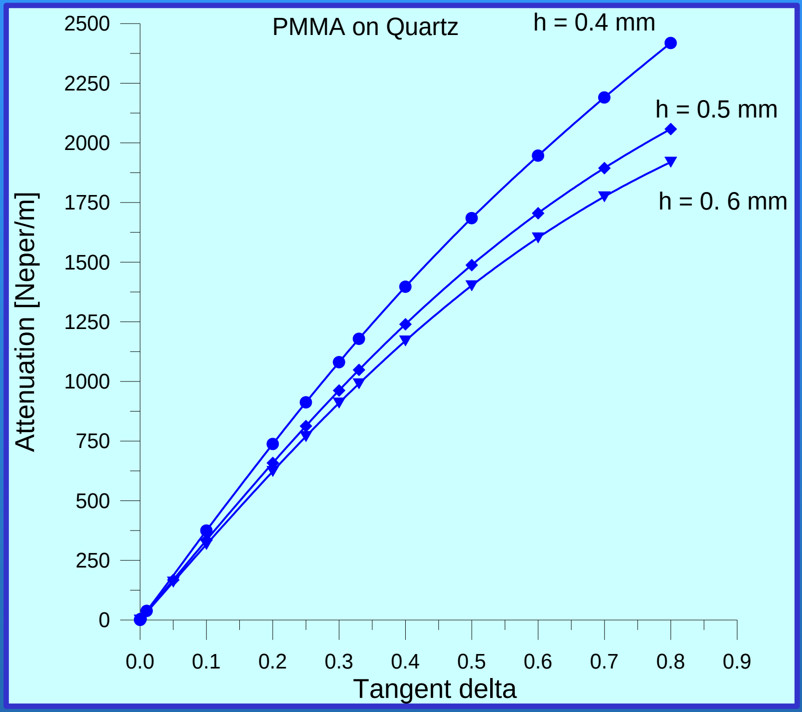


Fig.10. Attenuation α versus loss tangent of the surface layer, $f = 1\ MHz$.

Love wave phase attenuation dispersion curves

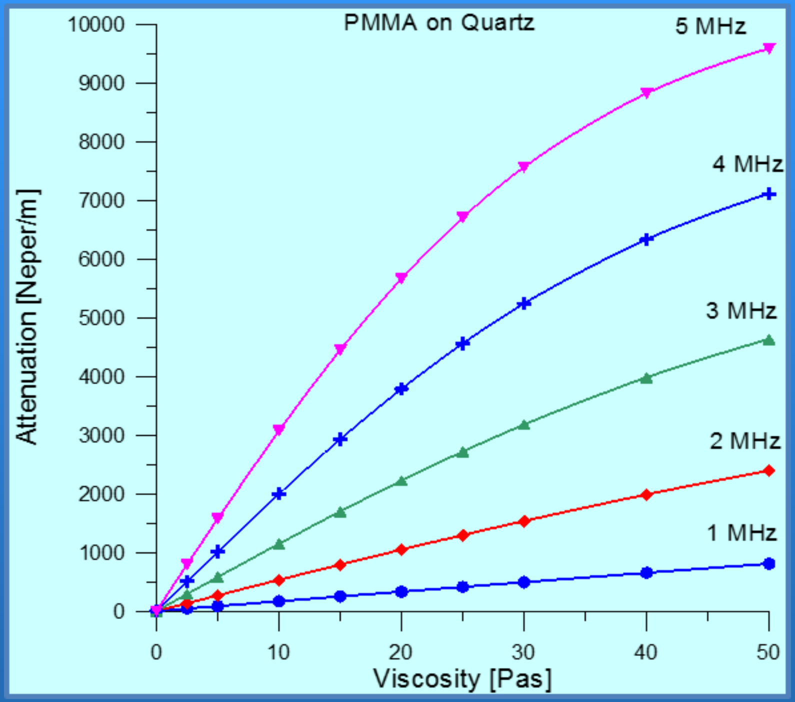


Fig.11. Attenuation α versus surface layer viscosity η_{44}, $h = 400\ \mu m$.

Conclusions

1. The results show that the Love waves can propagate in the investigated layered viscoelastic media

2. The impact of the rheological parameters on dispersion curves of the phase velocity and attenuation of Love wave was evaluated.
   Love wave phase velocity increases with increasing losses.

3. The Love wave sensor of the rheological parameters is electrically responsive. Owing to this fact, modern methods of the digital signal acquisition and processing can be efficiently used.

4. Proposed measuring setup can operate in real time and can be employed for measuring the rheological parameters under high pressure and temperature in the course of the technological processes.

Future works:

Determination of the rheological parameters (elasticity, viscosity, density) of viscoelastic media $G = G_1 + jG_2$.

Inverse Sturm-Liouville Problem:

Dispersion curves $\Rightarrow$ $\mu_B^0,\ \eta,\ \rho_B$

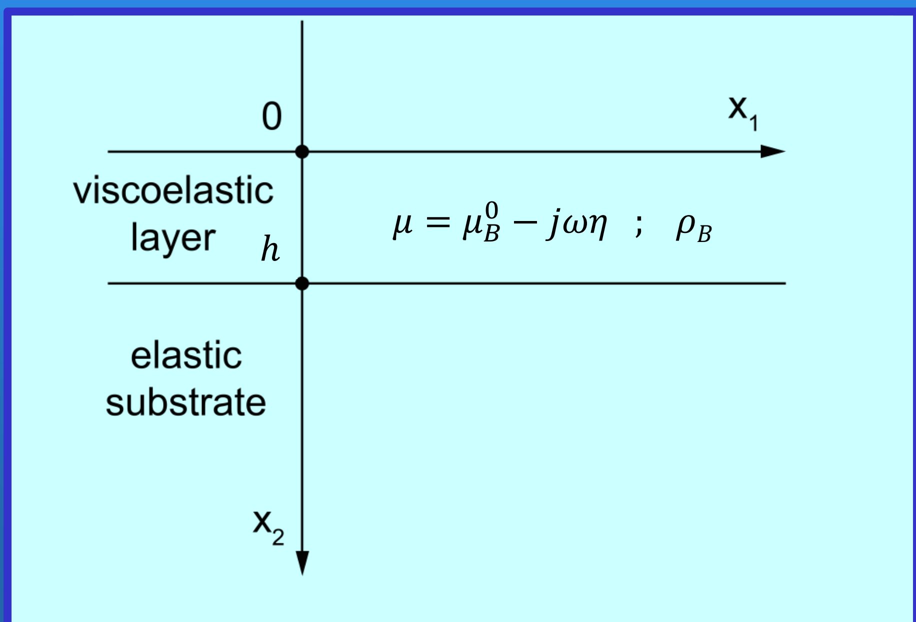


Fig.12. a) Viscoelestic layer over an elastic substrate

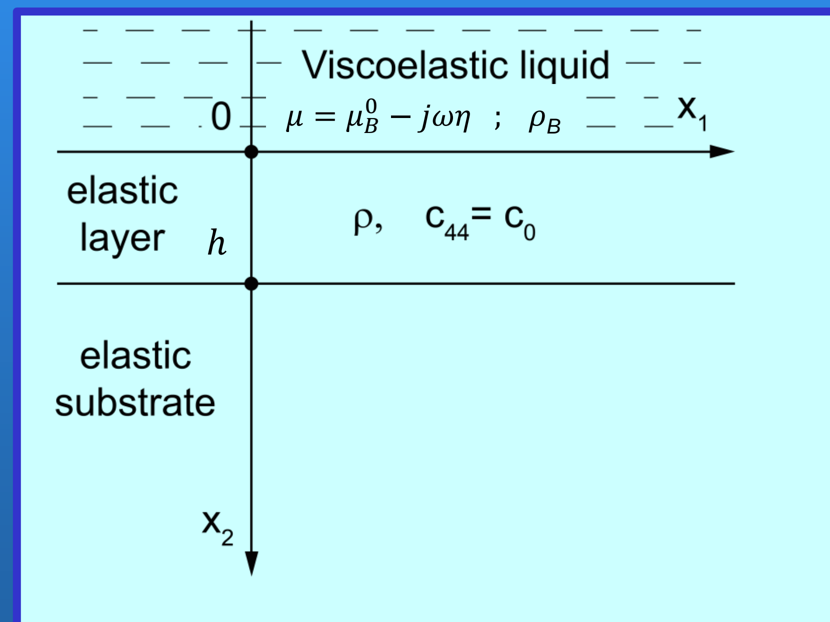


Fig.12. b) Viscoelastic liquid loading the waveguide surface

Potential applications of the Inverse Method

The results of the study can constitute the theoretical basis for application works in various branches of industries, namely:

1) in on-line investigation of liquid polymers during the course of technological processes (e.g., during processing of liquid polymers, during the pressurized encapsulation)

2) in on-line controlling of the viscoelastic properties of drilling fluids in petroleum and mining industries, during the oil and natural resources exploration

3) in the investigation of the viscoelastic properties of liquid food products (e.g., oils, fats, juices etc.).

4) in theory, design, and optimization of the ultrasonic sensors of the physical properties, chemo and biosensors, based on the use of surface Love waves

5) in geophysics and seismology. Investigation of the Earth interior.