„Determination of the elastic properties of thin layers using Love waves”

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- **Outline**

- 1) Introduction
- 2) New materials and new methods
- 3) Surface Acoustic Waves, Love waves
- 4) Generalized Love Waves
- 5) Direct (Sturm-Liouville) Problem
- 6) Experiment
- 7) Inverse problem
- 8) Determination of thin layers parameters
- 9) Conclusions

New Materials

- New Materials need New Measuring Methods to determine the mechanical parameters
- 1) Functionally Graded Materials
- 2) Composites
- 3) Intermetallics
- Elastic parameters are very important in engineering practice
- They are correlated with:
- 1) hardness
- 2) porosity
- 3) residual stresses
- 4) determine wear and exploitation characteristics

Classical methods for measuring the mechanical parameters of materials

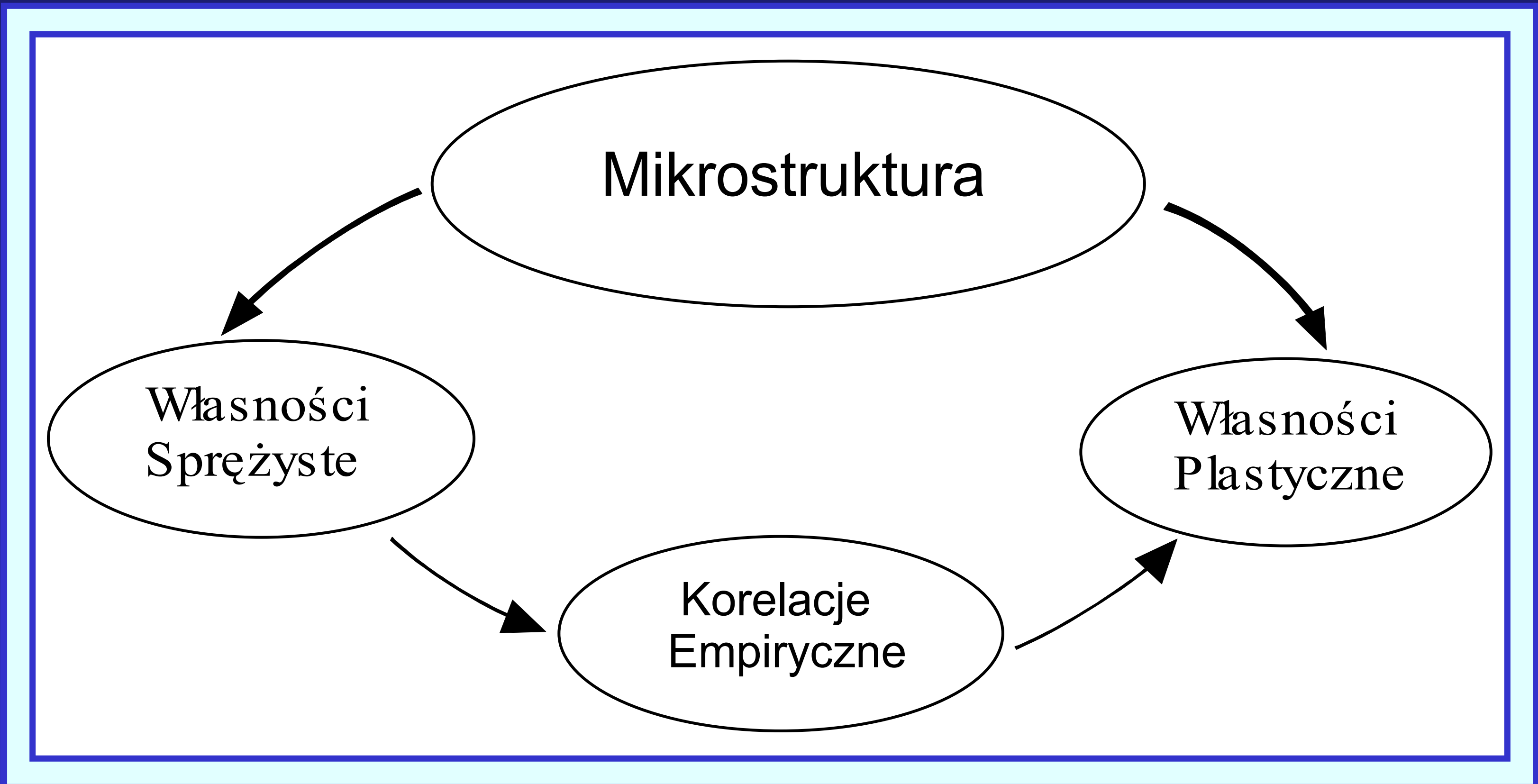
- 1) X-ray
- 2) Electron Microscopy
- 3) Electrochemical
- 4) Neutron scattering
- 5) Mechanical methods

- Disadvantages:
- 1) destructive
- 2) time consuming
- 3) tedious and expensive
- 4) can not be used “in situ”

Ultrasonic methods for determining the mechanical parameters of materials

- Using ultrasonic (bulk or surface) waves. The following mechanical parameters can be evaluated:
- 1) elastic and plastic coefficients
- 2) density and layer thickness
- 3) texture, hardness
- 4) cracks and delaminations
- 5) residual stresses
- Advantages:
- 1) nondestructive
- 2) can be used "in situ"
- 3) measurement in real time
- 4) can be computerized

Ultrasonic methods for determining the mechanical parameters of materials



Ultrasonic Surface Waves

- Surface acoustic waves are particularly convenient to investigate thin layers and graded materials
- Penetration depth is inversely proportional to frequency

  Higher frequency => lower penetration depth

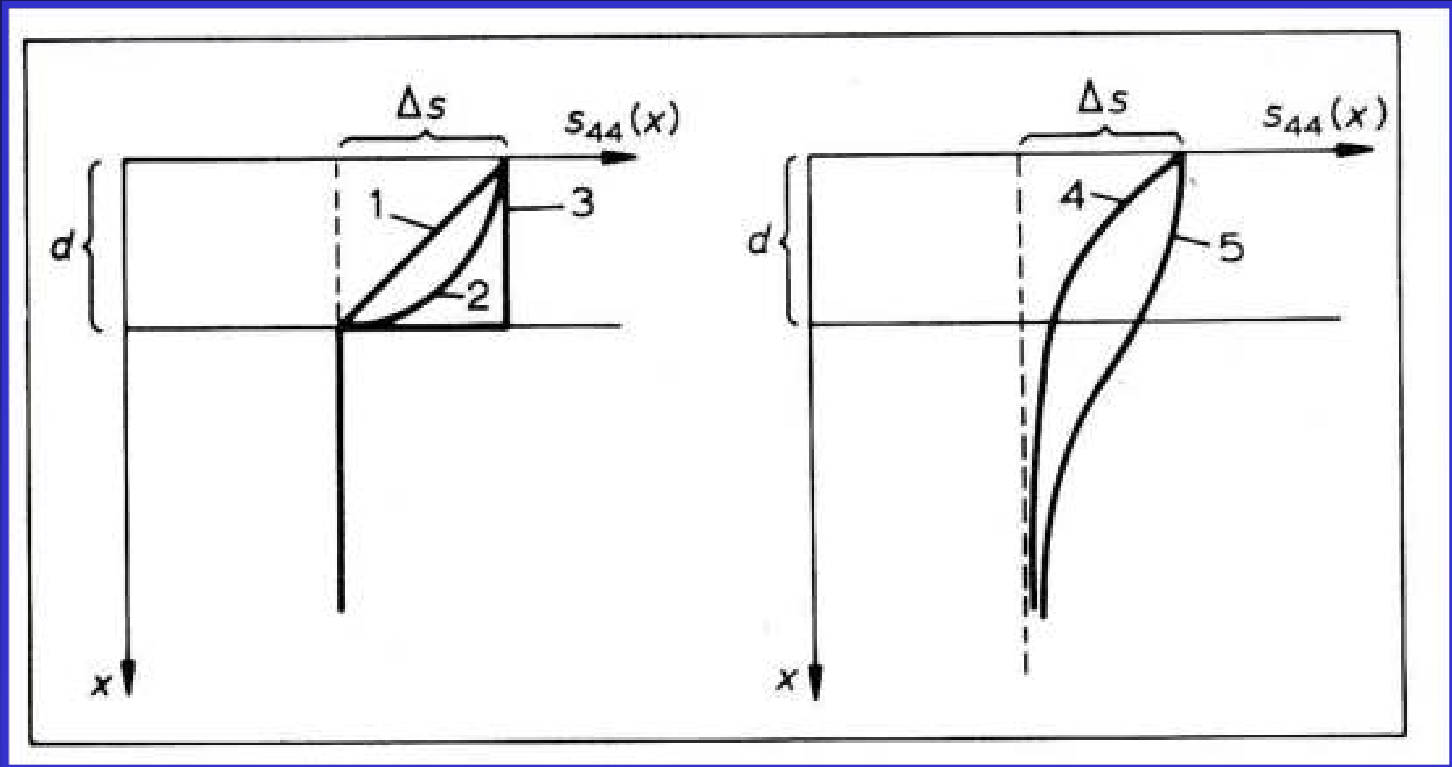
- 1) Rayleigh waves (longitudinal and shear vertical vibrations)
- 2) Love waves (shear horizontal vibrations)
- Advantages of Love waves:

  1) only one component of mechanical displacement

  2) simple mathematical description

  3) can be used in the viscosity sensors

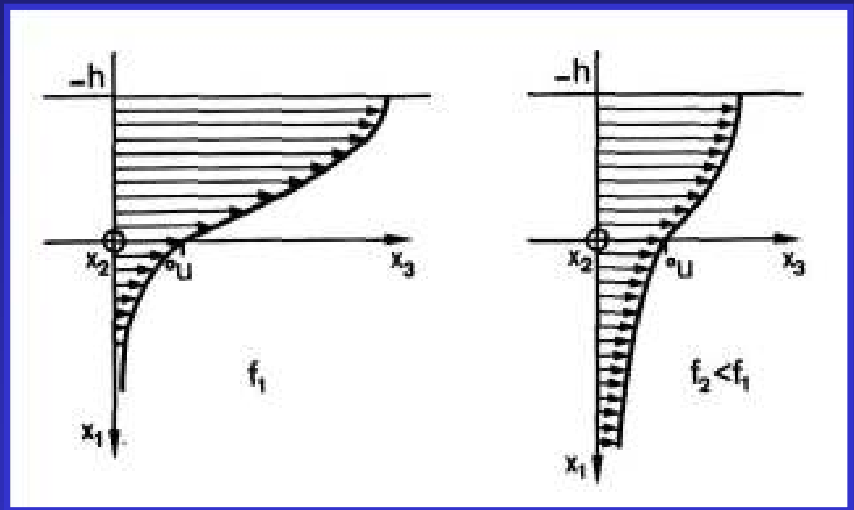
Profiles of shear compliance in (non-homogeneous) Graded Materials

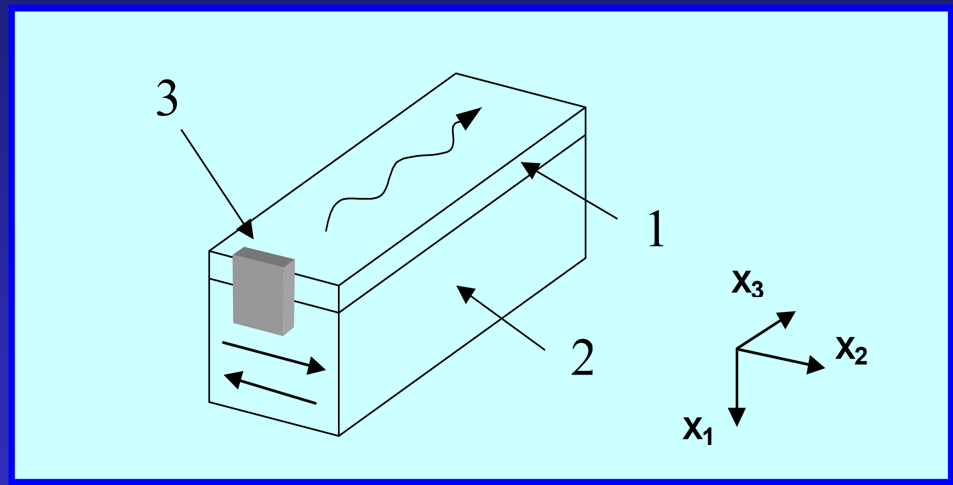


- **Fig.1. Changes of the shear compliance in function of depth, 1) linear, 2) quadratic, 3) step, 4) exponential, 5) Gaussian**

Love waves

- Distribution of mechanical displacement
- Velocity of the Love wave $v_L < v < v_S$





- Fig.1. Love wave amplitude in function of depth

Fig.2. Excitation of the Love wave, (3) PZT plate transducer, (1) Cu surface layer, (2) steel substrate

Love waves

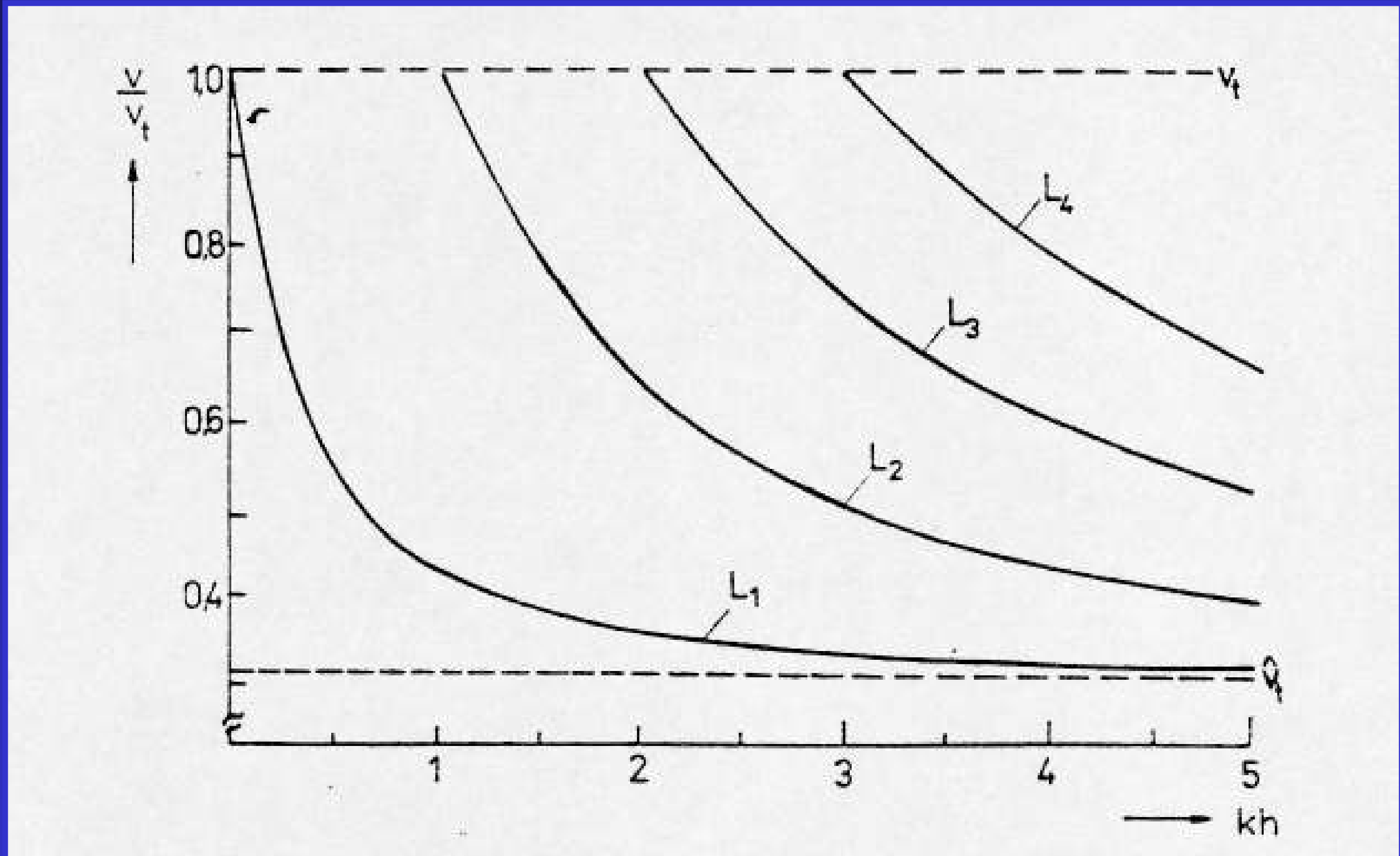


Fig.3. Dispersion curves

Generalized Love Waves (GLW)

- **Direct Sturm - Liouville Problem**
- **Forward Problem**

$$\frac{d}{dx}\left(c_{44}(x)\frac{df}{dx}\right)+\rho\omega^2 f = c_{44}(x)\,\beta^2 f \qquad \textbf{(1)}$$

$$\frac{df(0)}{dx}=0 \qquad ; \qquad f(\infty)=0 \qquad \textbf{(2)}$$

- **where : f(x) - amplitude of the GLW - eigenvector**

  $c_{44}(x)$ - distribution of elastic coefficient

  β^2 - propagation constant - eigenvalue

- **Mathematical model of Generalized Love Waves propagation in graded materials**

Sturm - Liouville Problem

- **Standing Waves : Travelling Waves**
  (Resonators)

Amplitudes

$B(x) \cdot \sin(\omega t)$ $\quad$ $f(z) \cdot \exp\left[j\left(\omega t - \beta x\right)\right]$

Differential Problems

$LB(x) = \omega^2 B(x)$ $\quad$ $L_1 f(z) = \beta^2 f(z)$

+ bound. cond. $\quad$ **+ bound. cond.**

- **where : L , L_1 - differential operators**

Eigenvalues

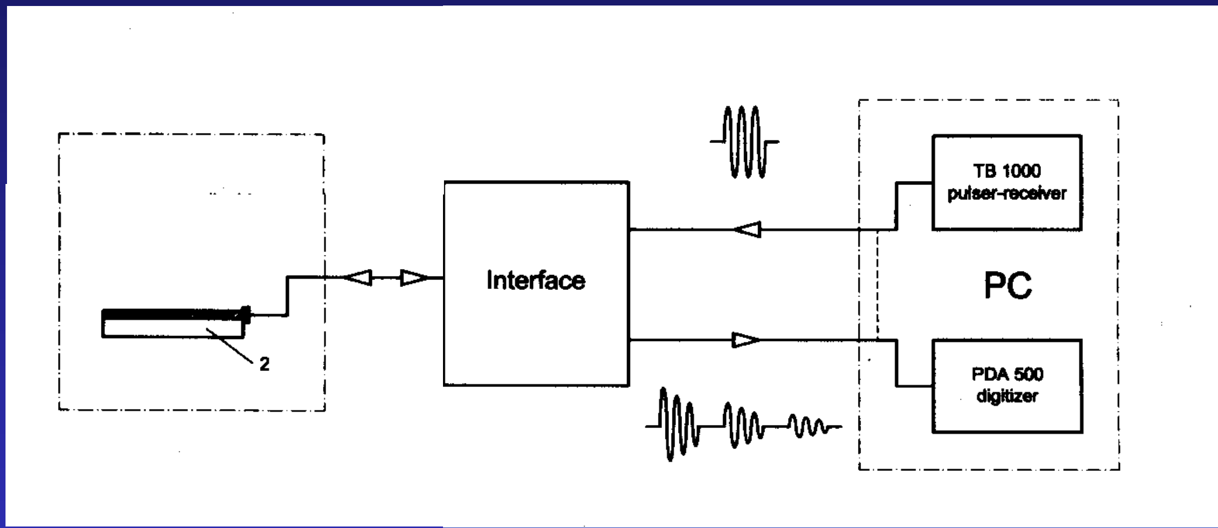
$\omega_1^2, \omega_2^2, \omega_3^2, \ldots, \omega_n^2$ $\quad$ $\beta_1^2, \beta_2^2, \beta_3^2, \ldots, \beta_n^2$

Eigenvectors

$B_1(x), B_1(x), B_1(x), \ldots, B_n(x)$ $\quad$ $f_1(z), f_2(z), f_3(z), \ldots, f_n(z)$

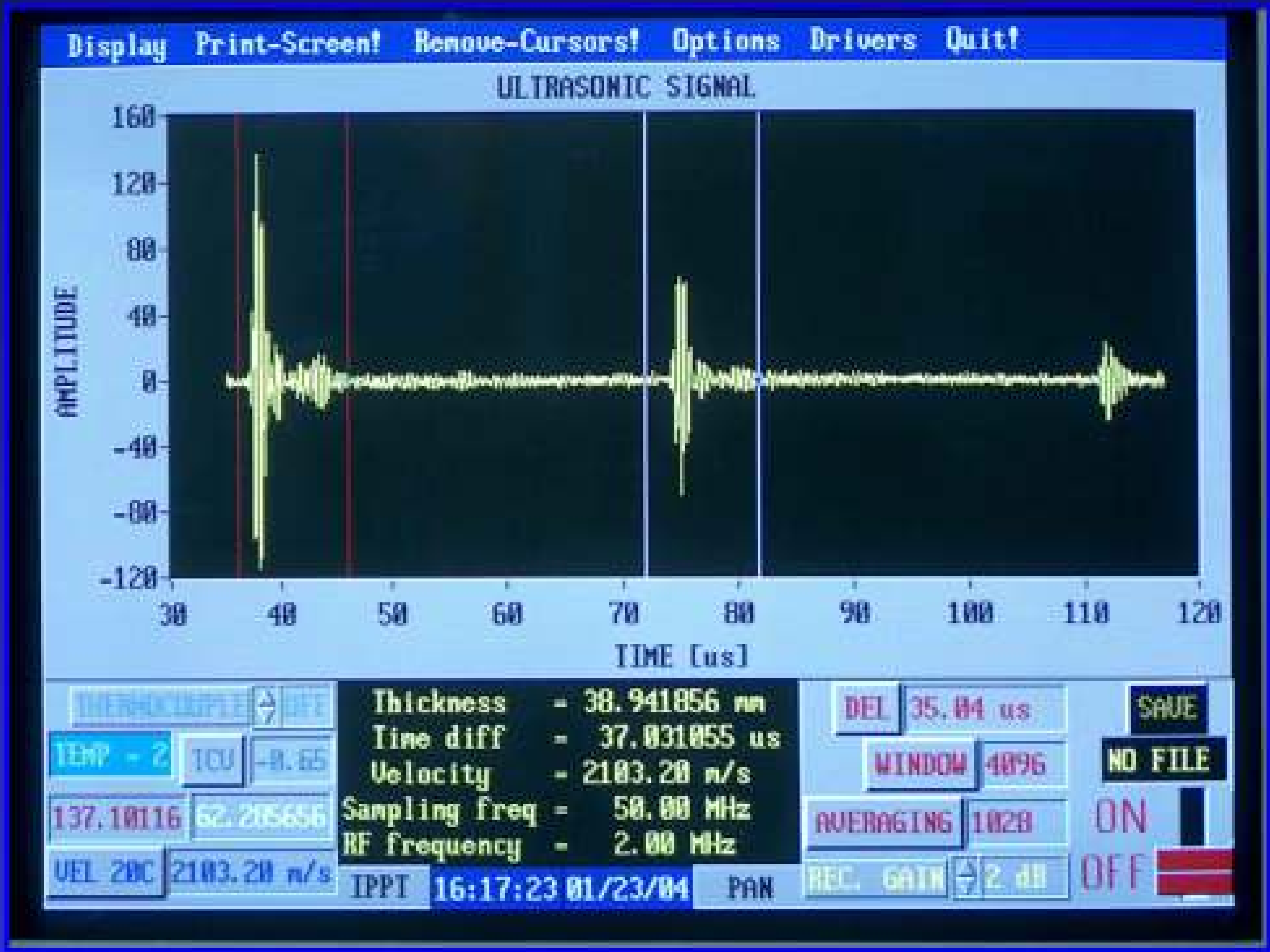
Experiment

- Measuring setup



- 2 - waveguide e.g., Cu on steel
- Time Of Flight (TOF) was measured by
  Cross Correlation method
  => velocity of the Love wave,
  accuracy = 0.2 %

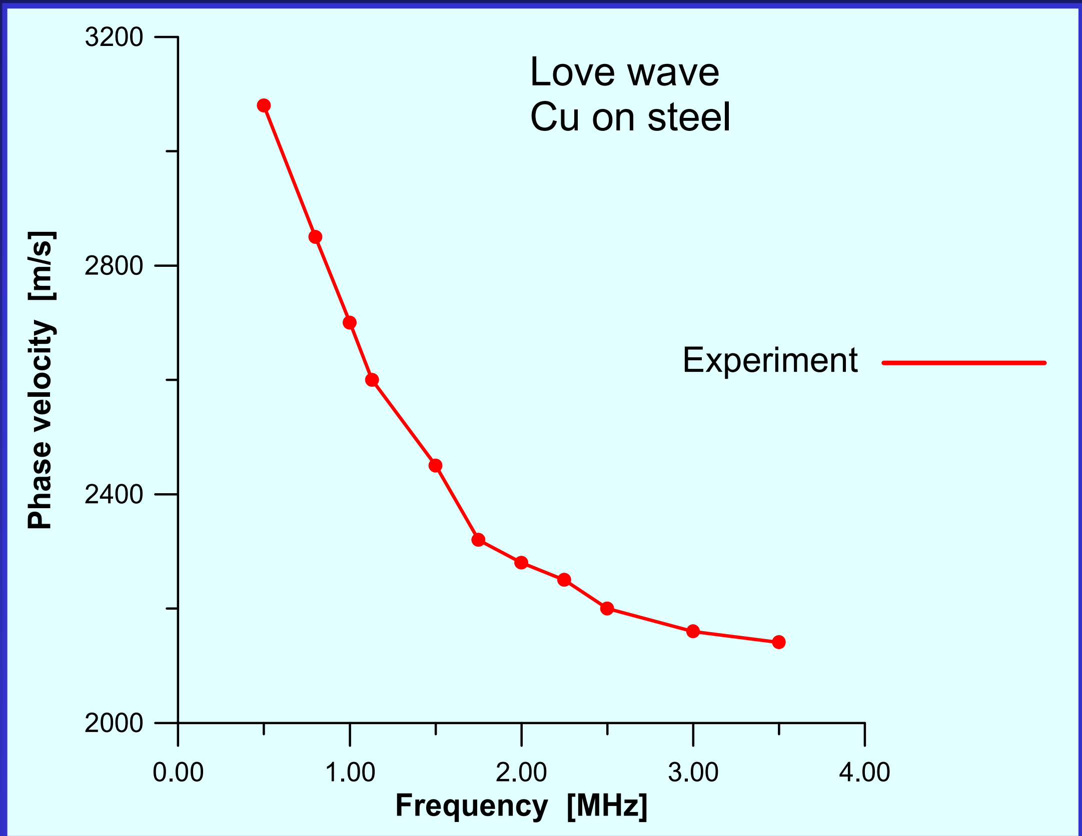
Experiment

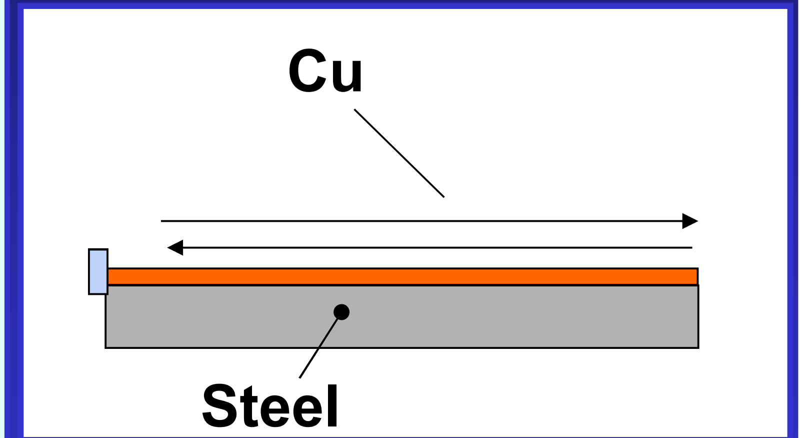


- **Fig.4. Time of flight (TOF) between two ultrasonic impulses (delimited by cursors) is evaluated by using the cross-correlation method.**

Dispersion curve

- **Sample: Cu on steel**

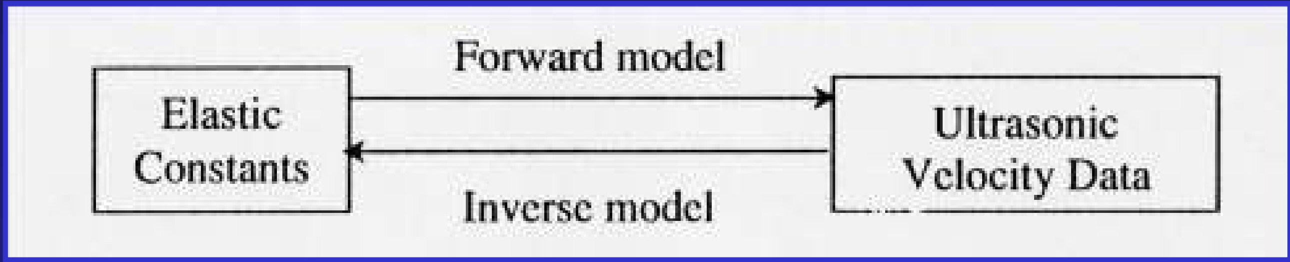




- **Fig.3. Measured dispersion curve. Cu layer on steel substrate.**

Inverse Methods

- Dispersion curves => elastic coefficients



- Information "a priori"
- To solve an Inverse Problem one should perform:

  1) Direct Problem

  2) Experiment

  3) Inverse analysis

- **Objective Function:** Π = Π (unknown parameters)

  Measure of the distance between the mathematical model of the object and real object
- Inverse Problem as an Optimization (Minimization) Problem

Example 1
Ceramic layer on a ceramic substrate

- **Dispersion equation:** $v = v(\omega)$

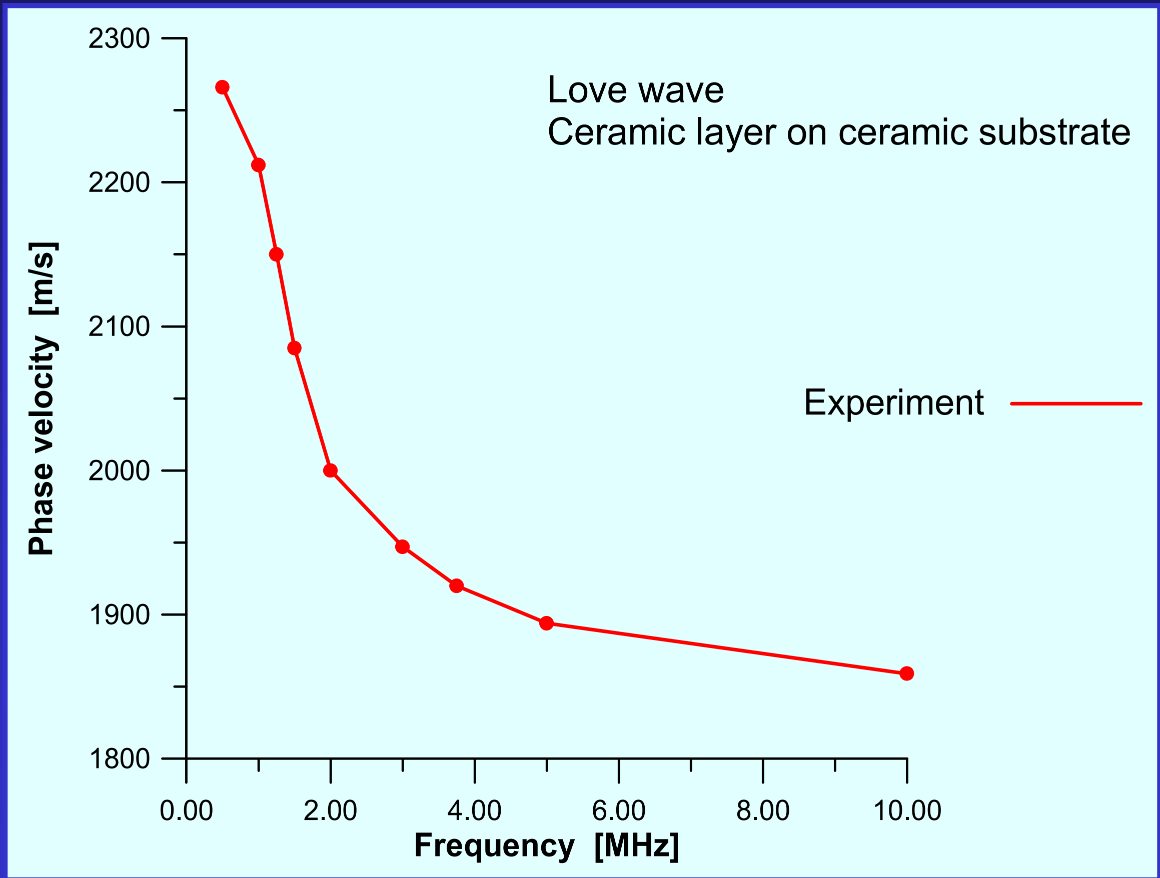
$$\Omega = \tan\left\{\sqrt{\left(\frac{v}{v_L}\right)^2 - 1} \cdot \beta h\right\} - \frac{c_{44S}}{c_{44L}} \frac{\sqrt{1-\left(\frac{v}{v_S}\right)^2}}{\sqrt{\left(\frac{v}{v_L}\right)^2 - 1}} = 0$$

- **Objective Function:**

$$\Pi = \sum_{j=1}^{N\exp} \left|\Omega\left(h, c_{44L}, \rho, \omega_j, v_j\right)\right|$$

- **Minimum of the objective function subjected to some constraints results in the optimal values of unknown parameters (e.g., thickness, elastic constants)**

Experimental Dispersion Curve
Ceramic layer on a ceramic substrate



- **Measured dispersion curve. Ceramic layer on a ceramic substrate**

Results of Example 1

- The following parameters were obtained from the Inverse Method:

  I. (h - unknown) $c_{44L} = 2.572e+10 \ N/m^2$

  starting point: h = 0 (exact value)

  constraints: $0 < h < 2e\text{-}3$ m; $h = 300\ \mu m$ - measured

  From Inverse Method: 1. h = 370 micrometers

- II. (h and c_{44L} - unknown)

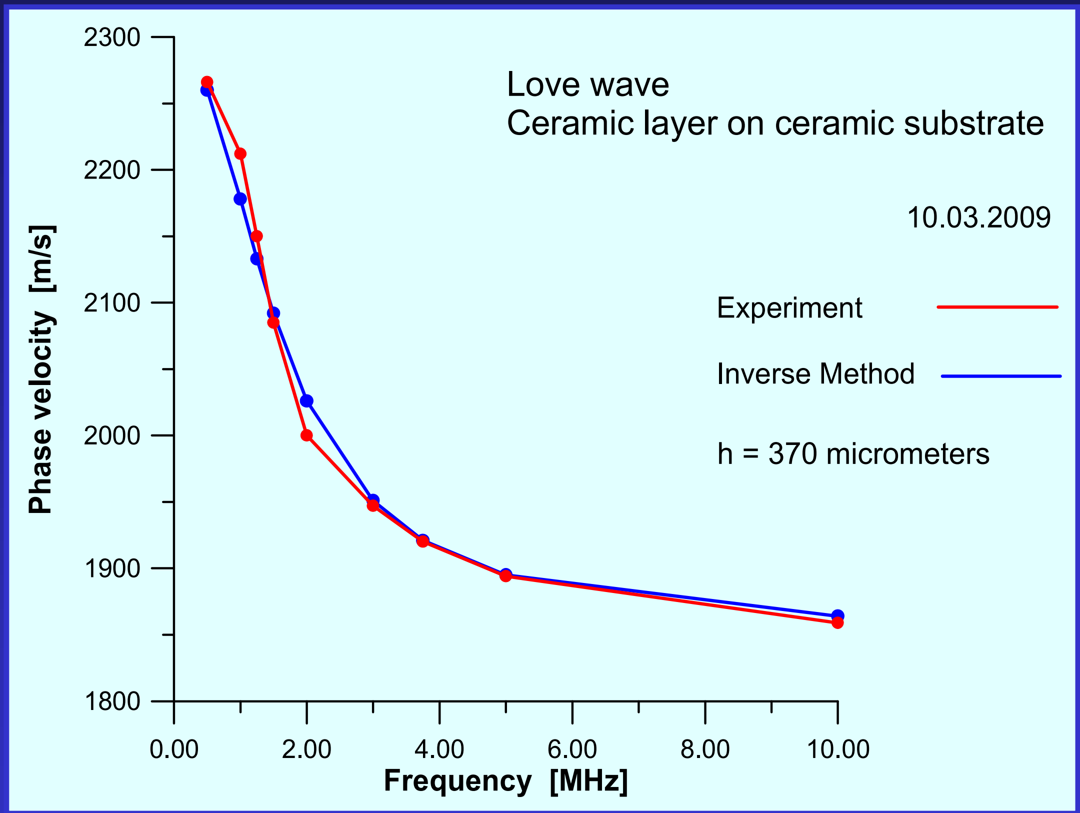
  starting point: h = 1e-3 m, c_{44L}= 0.3e+10

  constraints: $0 < h < 2e\text{-}3$ m; $1.5e+10 < c_{44L} < 3e+10$ (N/m^2)

  From Inverse Method:

  1. h=103 micrometers, c_{44L}= 2.3e+10 (N/m^2)

Verification (Example 1)



From Inverse Method

I. (h - unknown)

1. h=370 micrometers

(blue color)

2. Experimental curve

(red color)

- **Fig.4. Comparison of the experimental dispersion curve with that obtained from the Inverse Method.**

Example 2
Cu layer on steel substrate

- Dispersion equation relating phase velocity of the wave to frequency is the same as in Example1
- Objective function:

$$\Pi = \sum_{j=1}^{N\,\exp} \Omega^2 \left(h, c_{44L}, \rho, \omega_j, v_j\right)$$

- Minimization of the Objective Function

  subjected to some constraints results in the optimal values of unknown parameters (e.g., thickness, elastic constants)

Results of Example 2

- The following parameters were obtained from the Inverse Method: (exact value)

  I. (h - unknown) $c_{44L} = 3.925e+10 \ N/m^2$

  starting point: h = 0

  constraints: 0 < h < 2e-3 m; $h = 500 \ \mu m$ - measured

  From Inverse Method: 1. h = 541 micrometers

- II. (h and c_{44L} - unknown)

  starting point: h = 1e-4 m, c_{44L}= 3e+10

  constraints: 0 < h < 2e-3 m; 3e+10 < c_{44L}< 5e+10 (N/m^2)

  From Inverse Method:

  1. h=473 micrometers, c_{44L}= 3.766e+10 (N/m^2)

Results of Example 2

The following parameters were obtained from the Inverse Method:

- **III. (h, c_{44L} and ρ - unknown)**

  starting point: h = 1e-3 m, c_{44L}= 2e+10, ρ = 8e+3

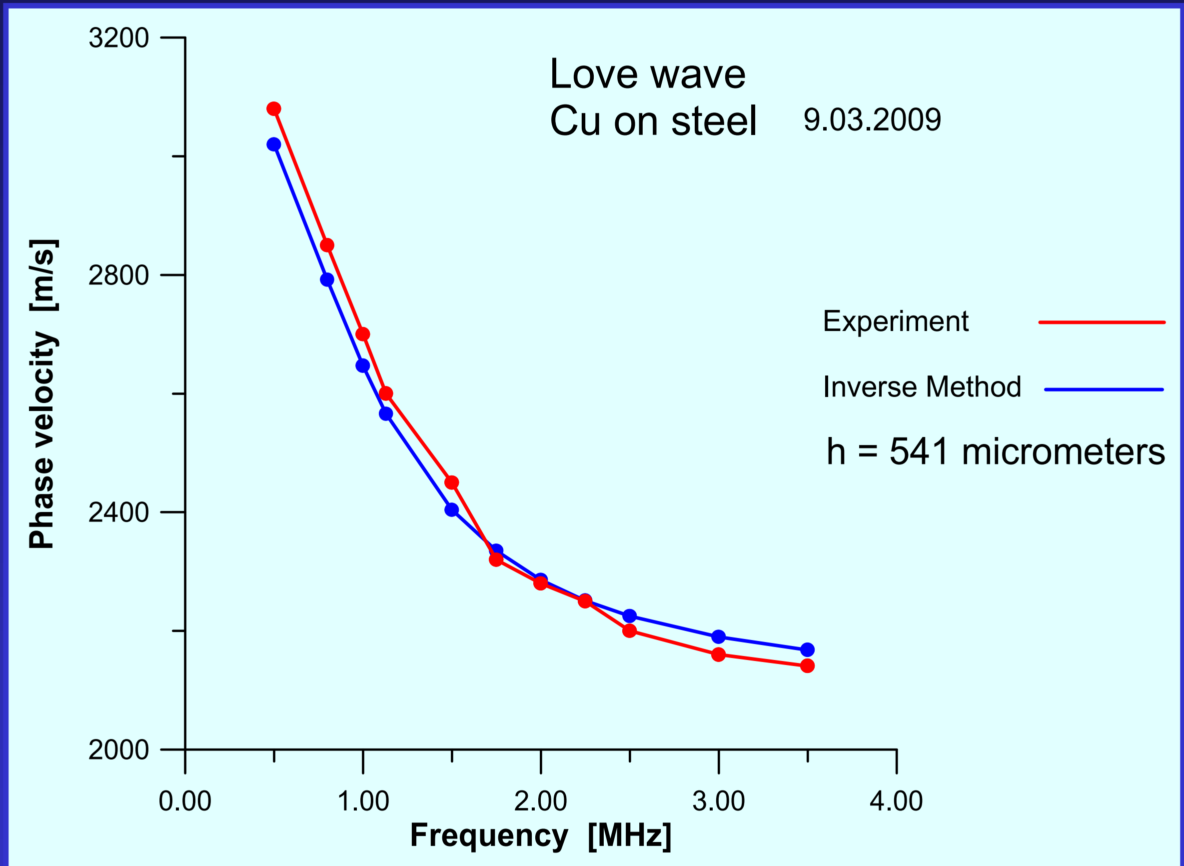
  constraints: 0 < h < 2e-3 m; 3e+10 < c_{44L}< 5e+10 (N/m^2)

  7e+3 < ρ < 9e+3

  From Inverse Method:

  1. h = 486 micrometers, c_{44L}= 3.828e+10 (N/m^2)
  ρ = 9e+3 kg/m^3
  Calculations were performed using Mathcad® program

Verification (Example 2)



From Inverse Method

I.

1. h= 541 micrometers (blue color)

2. Experimental curve (red color)

- **Fig.4. Comparison of the experimental dispersion curve with that obtained from the Inverse Method.**

Example 3 (in progress)

- **Continuous profile**
- **Steel sample subjected to the laser hardening**
- **Objective Function:**

$$\Pi = \sum_{j=1}^{N\exp}\left(v_j^{cal} - v_j^{\exp}\right)^2$$

- **Minimization of the objective function**

  v^{cal} are calculated from the direct S-L problem

  v^{exp} are measured for subsequent frequencies

- **Minimum of Π leads to a set (s_1, ... , s_{51}) that represents** $s_{44}(x)$

Conclusions

- Usefulness of the ultrasonic method employing Love Waves to investigate the elastic properties of thin layers was stated
- Future works:
- We plan to use of Laser Ultrasonic Techniques (LUT) to investigate the mechanical properties of materials
- Advantages of LUT over conventional ultrasonic techniques:

  1) is remote and non-contact

  2) broadband measurement

  3) high temperature measurement

  4) measurement in difficult access places