New Fascinating Properties and Potential Applications of Love Surface Waves

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September 16th 2021, IEEE, IUS, Xi'an, CHINA.

Polish Academy of Sciences

69 Institutes, 3700 Scientists (Researchers)

Institute of Fundamental Technological Research:

300 employees, 200 Researchers

Main domains of research:
- Advanced Materials
- Ultrasonics,
- Biotechnology
- Mechanics
- Applied Mathematics

Laboratory of Acoustoelectronics:

- Surface acoustic wave sensors employing Love and Bleustein-Gulyaev surface waves
- Mathematical modeling and numerical methods
- Computerized instrumentation
- High pressure characterization of liquids using Love and BG surface waves

Outline of the Presentation

- 1) Discovery of Love waves
- 2) Unique properties of Love waves
- 3) Analogies between Love waves and
  a) Electromagnetism
  b) Quantum mechanics
- 4) Mathematical modeling of Love wave propagation:
  Direct Sturm – Liouville Problem
- 5) New analytical formulas for the mass sensitivity of Love wave sensors
- 6) Unexpected counter intuitive phenomena in Love wave waveguides
- 7) New mathematical tools and Perspectives for Love wave sensors

How Love waves look like?

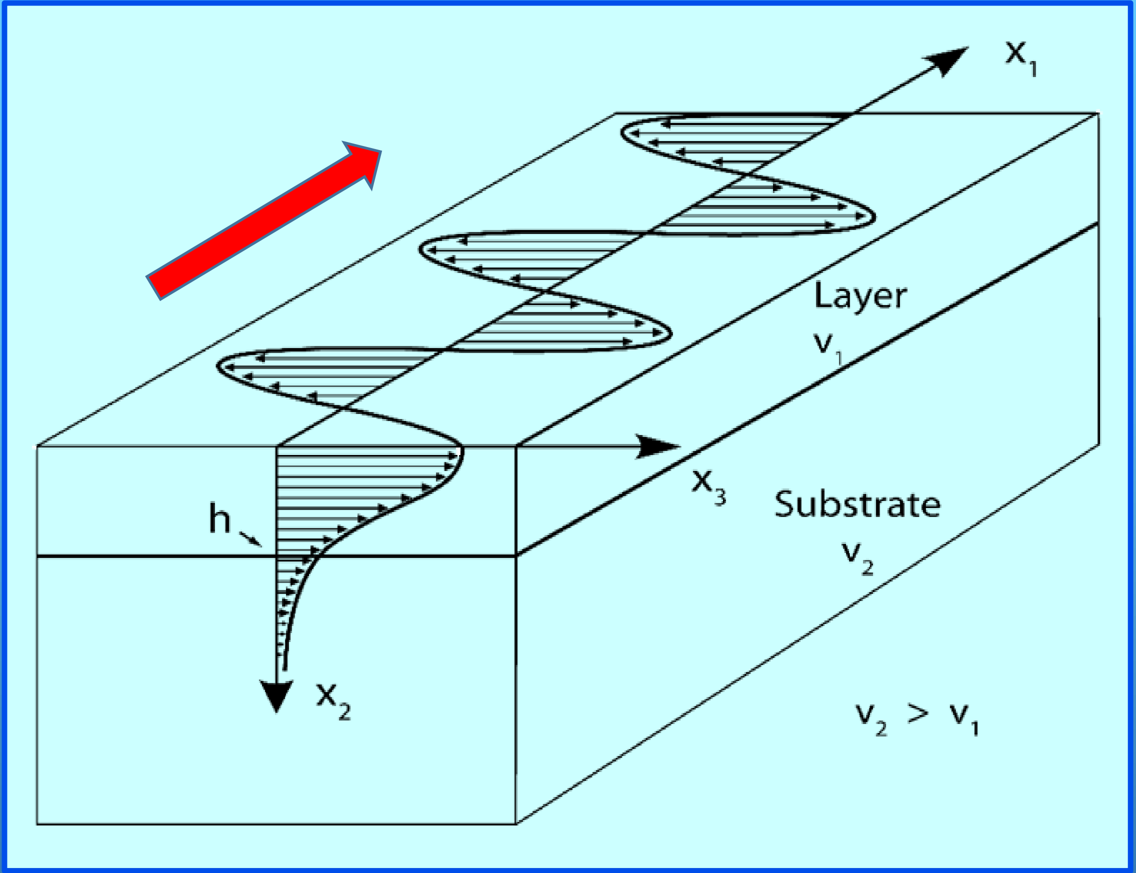


Fig.1.

In the direction of the axis x_2 (depth) Love waves are standing waves

In the direction of the axis x_1 Love waves are traveling waves

Love waves have only one SH component of the mechanical displacement u_3 - along the x_3 axis

$$u_3(x_1, x_2, t) = f(x_2) \cdot exp[j(\beta x_1 - \omega t)]$$

Deadly Love waves are generated during earthquakes.
Benign Love waves are employed in: 1) sensors and 2) non-destructive testing (NDT).
Love waves spans the frequency range from 0.001 Hz (seismic) to ~ 10 GHz (sensors).

Historical perspective for Love surface waves

1885 – Rayleigh waves (surface waves)

1911 – Love waves (SH surface waves)

1917 – Lamb waves (plate waves)

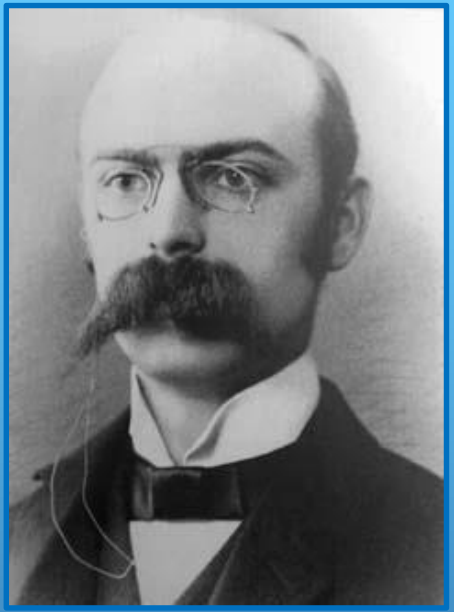
1924 – Stoneley waves (solid-solid interface waves)

1927 – Sezawa waves (plate waves)

1947 – Scholte waves (liquid-solid interface waves)

1968 – Bleustein-Gulyaev waves , (SH surface waves in piezoelectrics)

Fig.2.



Augustus, Edward, Hough Love - 1911

Why Love waves are special?
Unique properties of Love waves

Love surface waves have many unique features that differentiate them from other types of surface waves, such as: Rayleigh, Lamb and/or Stoneley waves.
For example, Love surface waves:

1. have only one shear horizontal (SH) component of vibration (mechanical displacement)
2. have mathematical model with a moderate complexity
3. have exact analogues in electromagnetism and integrated optics (TM modes in planar dielectric waveguides)
4. have a direct analogy in quantum mechanics (quantum particles in potential wells)
5. Love wave can be regarded as a representative of electromagnetic waves among mechanical waves
6. The relative simplicity of the mathematical model of the Love wave allows us to achieve a number of useful analytical formulas, e.g., the formula for the mass sensitivity $S_{\sigma}^{v_p}$ of the Love wave sensor

Love wave as a mechanical (elastic) Shear Horizontal (SH) surface wave

Love wave is the simplest (SH surface) mechanical wave.
Love wave has only one component of the mechanical displacement u_3:

$$u_1 = u_2 = 0; \quad u_3 = u_3(x_1, x_2)$$

$$x_1 = \mathrm{x}; \quad x_2 = y; \quad x_3 = z$$

Strain:

$$\begin{bmatrix} 0 & 0 & \epsilon_{13} \\ 0 & 0 & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & 0 \end{bmatrix} \quad (1)$$

Antiplane stress:

$$\begin{bmatrix} 0 & 0 & \sigma_{13} \\ 0 & 0 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & 0 \end{bmatrix} \quad (2)$$

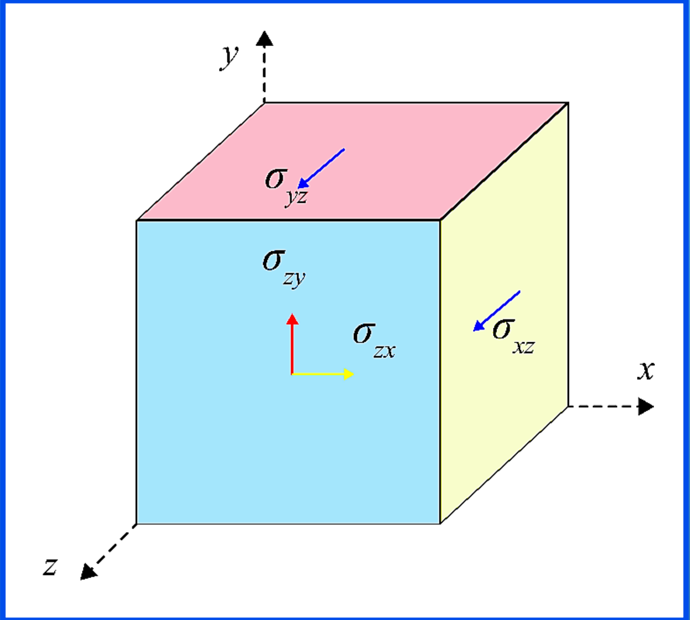


Fig.3. Elementary cube

$$\sigma_{\mathrm{zx}} = \sigma_{13}; \quad \sigma_{\mathrm{z}y} = \sigma_{23}$$

Mathematical description of the Rayleigh surface wave is significantly more difficult than that of the Love wave.

Love surface waves have an exact analogue in electromagnetism (TM guided waves in dielectrics)

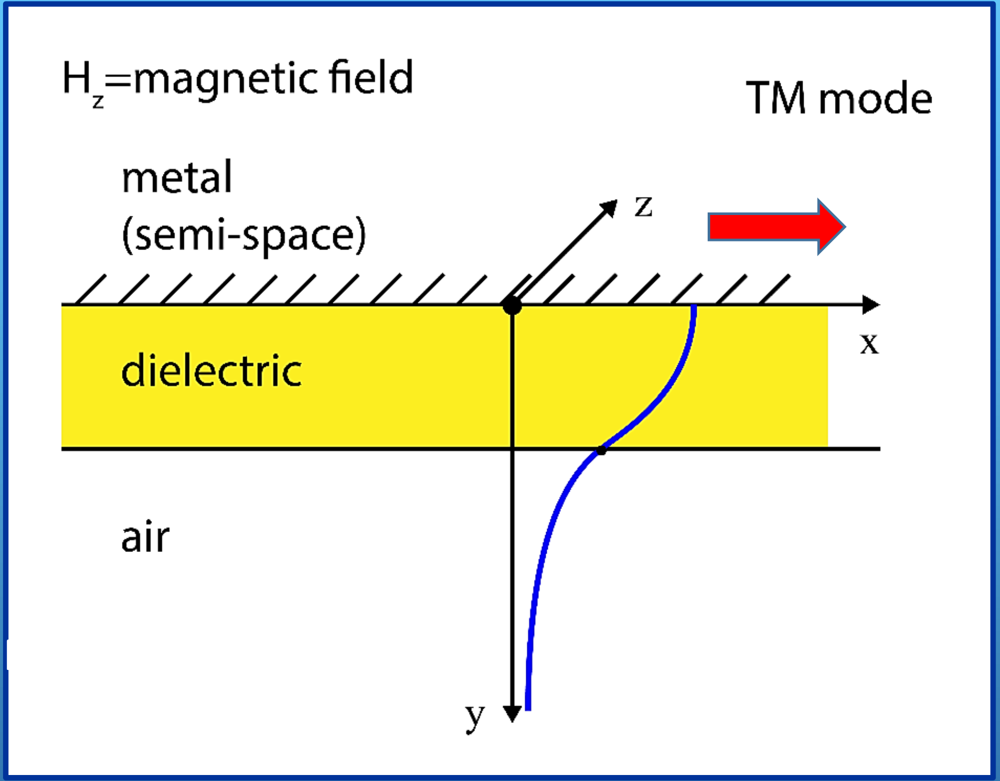
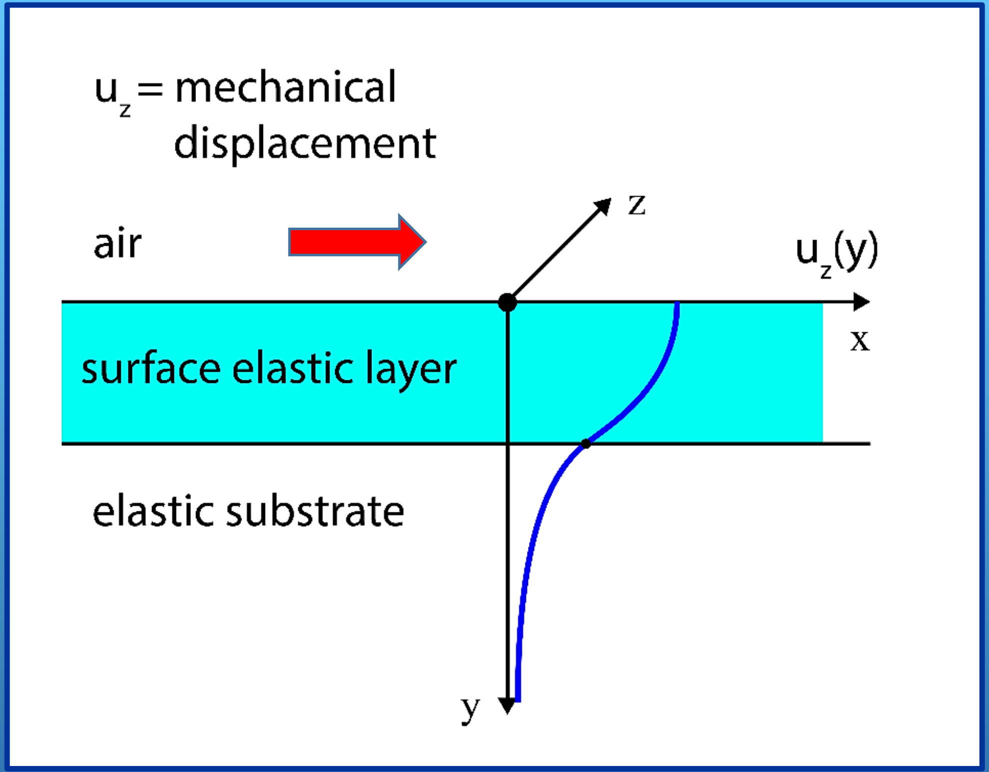


Fig.4.

Fig.5.



S.S. Attwood, Journal of Applied Physics, 1951: Microwave engineering

A.H.E. Love, Some Problems in Geodynamics, 1911: Seismology

Acoustics preceded electromagnetism by 40 years !!!

$$u_z(x, y, t) = f(y) \cdot exp[j(\beta x - \omega t)]$$

TM modes, ELECTROMAGNETISM

Maxwell equations:

1. Dielectric layer (slab) – TM modes

$$\frac{d^2H_z}{dy^2} + \mu\varepsilon\omega^2 \cdot H_z = \beta^2 \cdot H_z \qquad (3)$$

2. Boundary conditions

a) $\frac{dH_z}{dy} = 0$ at $x = 0$ (4)

b) continuity of $\frac{dH_z}{dy}$ and H_z at $x = h$ (5)

3. Solutions:

$$H_z(x,y,t) = A \cdot \cos(q_y y) \cdot exp[j(\beta_x x - \omega t)] \qquad (6)$$

where: $q_y = \sqrt{\beta^2 - \mu\varepsilon\omega^2}$

Love waves, THEORY OF ELASTICITY

Equations of motion:

1. Elastic surface layer

$$\frac{d^2u_z}{dy^2} + \frac{\rho}{c_{44}}\omega^2 \cdot u_z = \beta^2 \cdot u_z \qquad (7)$$

2. Boundary conditions

a) $\frac{du_z}{dy} = 0$ at $x = 0$ (8)

b) continuity of $\frac{du_z}{dy}$ and u_z at $x = h$ (9)

3. Solutions:

$$u_z(x,y,t) = B \cdot \cos(q_y y) \cdot exp[j(\beta_x x - \omega t)] \qquad (10)$$

where: $q_y = \sqrt{\beta^2 - \frac{\rho}{c_{44}}\omega^2}$

To find distributions of $u_z(y)$ and $H_z(y)$ it is necessary to solve equations of motion

Eqs. 3, 7 = Eigenequations for eigenvalues and eigenfunctions

Analogy of Love waves in Integrated Optics: (Optical Planar Waveguides)

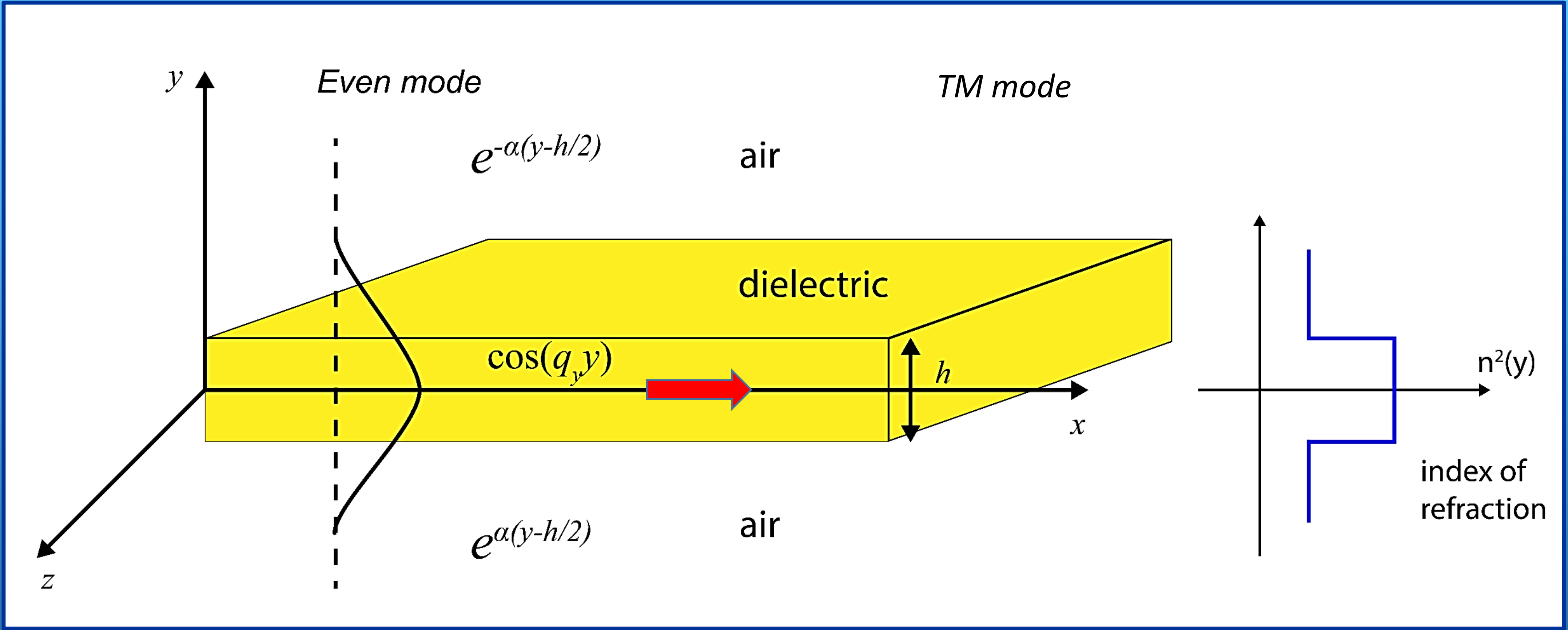


Fig.6.

The fundamental TM mode in the optical planar waveguide (see Fig.6) can be regarded as an optical analogue of the Love wave

Theory of this type of planar optical waveguides was developed in 1960s

Mathematical methods of modern Optics are very advanced and can be transferred to Acoustics

Analogy of Love waves with Quantum Mechanics

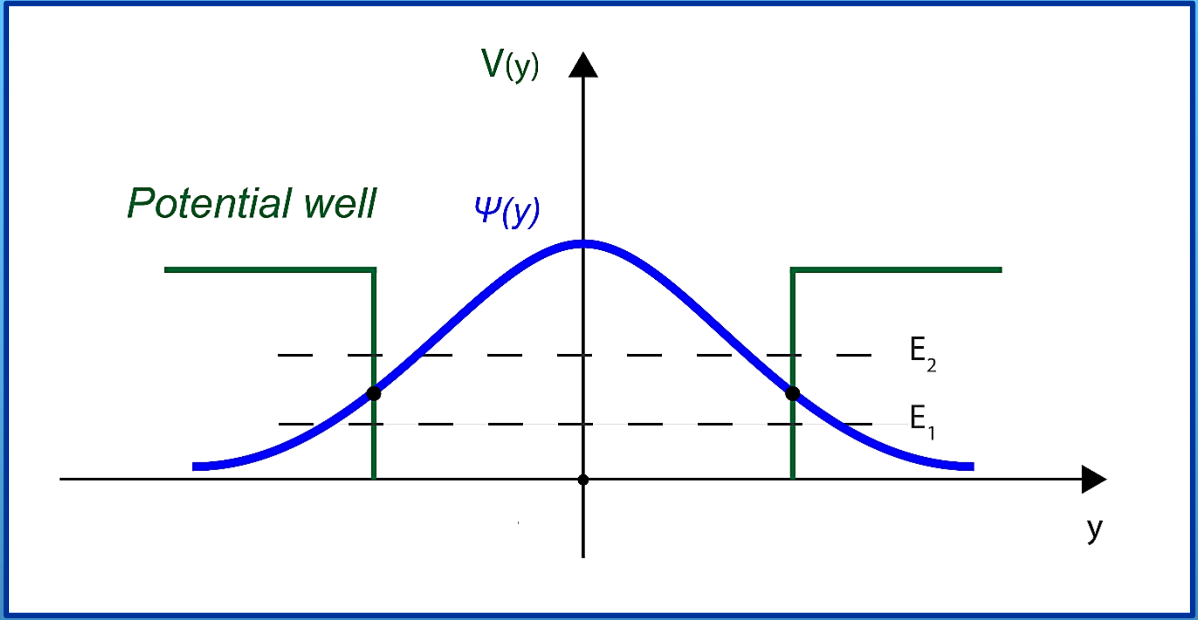


Fig.7.

Schrödinger equation (1926)
$\Psi(y)$ – Wave function
$|\Psi(y)|^2$ – Probability density
$E_1,\ E_2$ – Energy levels
Direct Sturm- Liouville Problem

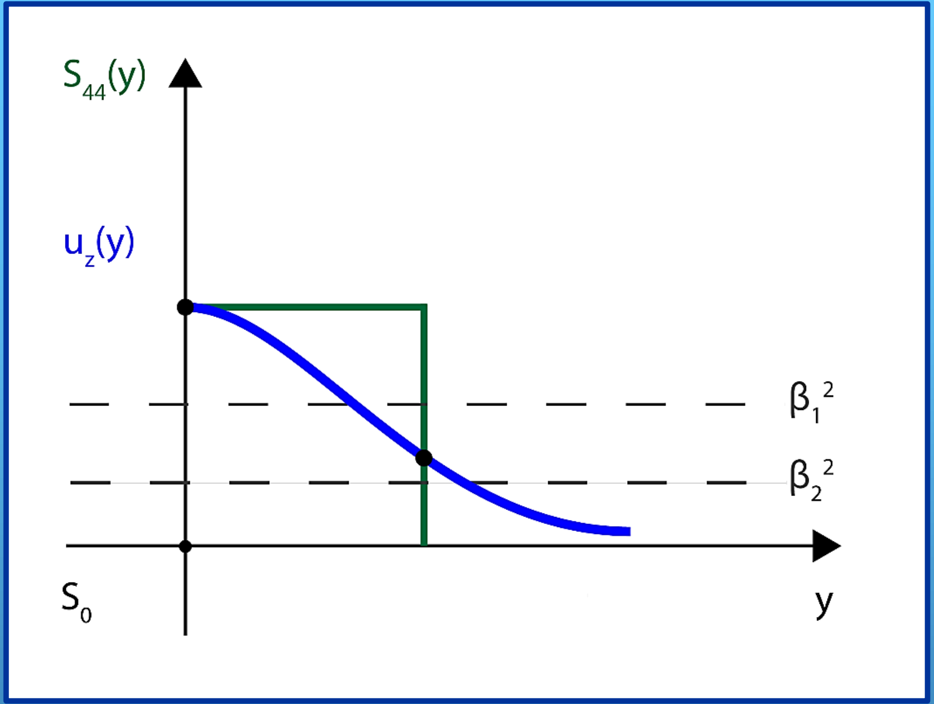


Fig.8.

Equation of motion (Newton laws)
$u_z(y)$ – Mechanical displacement
$|u_z(y)|^2$ – Energy density
$\beta_1^2,\ \beta_2^2$ – Eigenvalues (Propagation constants)
Direct Sturm- Liouville Problem

Mathematical methods developed in Quantum Mechanics are very sophisticated, original and rich.

Mathematical methods used in Quantum Mechanics may be one day transferred into the theory of Love waves

PARTICLE IN QUANTUM WELL

Schrödinger equation:

1. Quantum well

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi}{dy^2} + V\cdot\Psi = E\cdot\Psi \qquad (11)$$

2. Boundary conditions:

b) continuity of $\frac{d\Psi}{dy}$ and Ψ at $x = \pm h$ (12)

3. Solutions:

$$\Psi(x,t) = A\cdot\cos(q_y y)\cdot exp[j(-\omega t)] \qquad (13)$$

where: $q_y = \sqrt{E - V}$

LOVE WAVES

Equations of motion:

1. Elastic surface layer

$$\frac{d^2u_z}{dy^2} + \frac{\rho}{c_{44}}\omega^2\cdot u_z = \beta^2\cdot u_z \qquad (14)$$

2. Boundary conditions:

a) $\frac{du_z}{dy} = 0$ at $x = 0$

b) continuity of $\frac{du_z}{dy}$ and u_z at $x = h$ (15)

3. Solutions:

$$u_z(x,y,t) = B\cdot\cos(q_y y)\cdot exp[j(\beta_x x - \omega t)] \qquad (16)$$

where: $q_y = \sqrt{\beta^2 - \rho s_{44}\omega^2}$

$s_{44} = 1/c_{44}$

Equations describing many phenomena in different domains of physics have exactly the same form

The same differential equations with identical boundary conditions have certainly the same solutions

Direct Sturm-Liouville Problem
Common Mathematical Model
of 3 different physical phenomena

Operator L

eigenvector

eigenvalue

$$\left[\frac{1}{c_{44}}\frac{d}{dx_2}\left(c_{44}(x_2)\frac{d}{dx_2}\right)+\frac{1}{c_{44}}\rho\omega^2\right]f(x_2)=\beta^2 f(x_2) \qquad (17)$$

$$Lf(x_2) = \beta^2 f(x_2)$$

$$\frac{df(0)}{dx_2}=0 \qquad f(\infty)=0 \qquad (18)$$

$$\left(\beta_j^2, f_j(x_2)\right)\Big|_{j=0,1,\,\dots}$$

- Sturm-Liouville Problem (17-18) for eigenvalues and eigenvectors:
- β^2 = eigenvalue – determines the phase velocity of the Love wave ($\beta=\omega/v_P$ - wave number)
- $f(x_2)$ = eigenvector – determines the distribution of the mechanical displacement with depth
- It is amazing that: Common mathematical model describes:
- a) Love surface waves propagation

$$u_3(x_1,x_2,t)=f(x_2)\cdot exp[j(\beta x_1-\omega t)]$$

- b) motion of quantum particles in a potential well (Schrödinger equation)
- c) planar optical waveguides

Mathematics is certainly the Queen of Sciences

Physical Implementation of Love Wave Sensors

Why Love waves are so successful in sensors, biosensors and chemosensors?

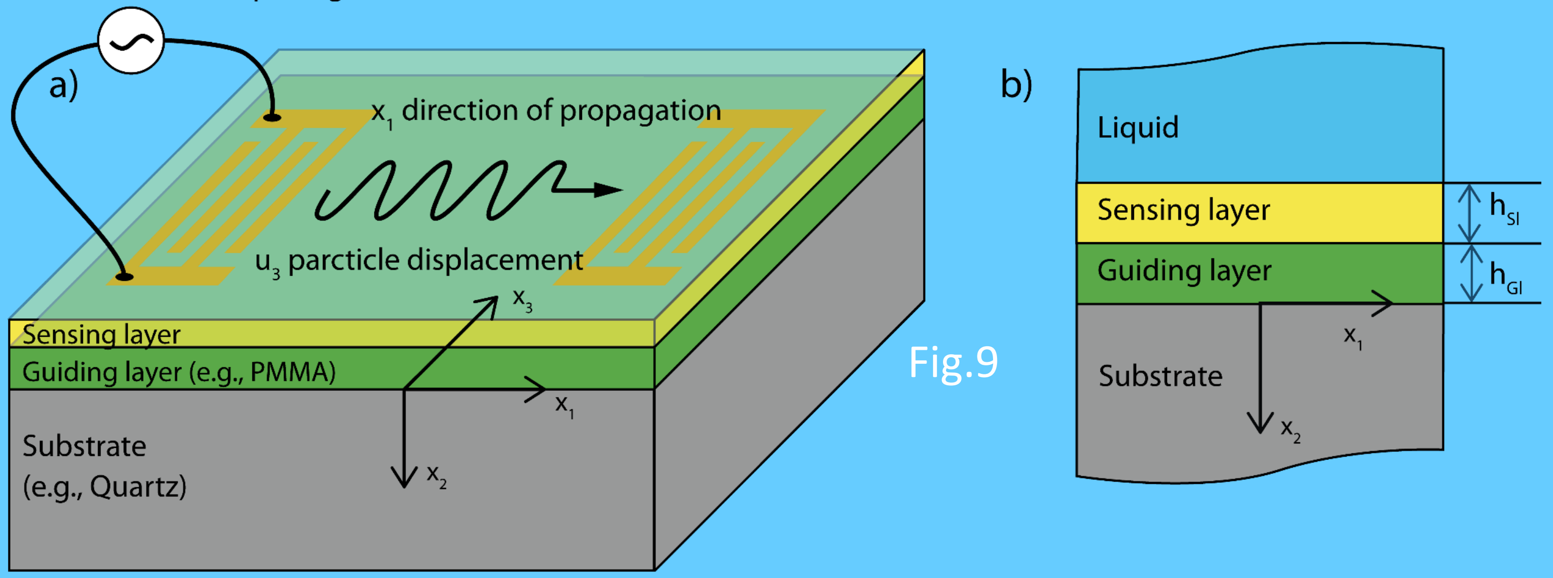


Fig.9

a) Typical dimensions - 1 x 5 x 20 mm
b) Circuit configuration - resonator or delay line
c) Frequency range - 50 - 500 MHz
d) Wavelength range - 10 - 100 µm

1. Love waves as Shear Horizontal waves can operate in liquid environment
2. High concentraction of Energy in the vicinity of the waveguide surface → high sensitivity of the sensor
3. Simple construction of Love wave waveguides
4. Love wave sensors have the highest mass sensitivity of all sensors that use other types of waves: e.g., Rayleigh, Lamb, SH plate, flexural plate waves etc.

Love waves are essentially mechanical waves, therefore the change of the mechanical conditions on the surface of the waveguide (e.g. in the sensing layer) will affect the velocity and attenuation of the Love wave

First Publications on Love Wave Sensors

First attemps to employ Love waves into sensors were carried out at the Polish Academy of Sciences, exactly 70 years after the discovery of A.E.H. Love (1911)

1) P. Kiełczyński and R. Płowiec, Polish Patent (1981)

P. Kiełczyński, W. Pajewski, European Mechanics Colloquium (1987), (Nottingham, Great Britain)

3) P. Kiełczyński, W. Pajewski, IEEE Ultrasonic Symposium, (1988), (Chicago, USA)

4) P. Kiełczyński, R. Płowiec, Journal of the Acoustical Society of America, JASA - (1989):

In these papers we have developed a theoretical (perturbation) model of the Love wave sensors along with its experimental verification

This model is the basis for the operation of a) biosensors, b) chemosensors and c) sensors of physical quantities

The first similar papers on Love wave sensors appeared in the USA 3 years later:

5) G. Kovacs et al., IEEE Ultrasonic Symposium (1992)

6) A. Venema et al., Applied Physics Letters, (1992)

7) M.J. Velekoop et al., IEEE Ultrasonic Symposium (1994)

8) E. Gizeli et al., IEEE Trans on UFFC, (1992)

Determination of the shear impedance of viscoelastic liquids using Love and Bleustein–Gulyaev surface waves

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(Received 3 May 1988; accepted for publication 22 February 1989)

This paper presents a new method, using shear *SH* (shear horizontal) surface waves in solids, to determine the rheological parameters of viscoelastic liquids. Appropriate analytical formulas have been derived for Love and Bleustein–Gulyaev surface waves. The sensitivity of the proposed method is compared to that of the classical Mason method employing *SH* bulk waves. The measuring range of the proposed and classical methods is discussed in detail. Preliminary measurements are performed for typical mineral oil. The measured quantities agree very well with those obtained theoretically. The proposed method can be a few orders of magnitude more sensitive than the bulk wave method.

PACS numbers: 43.20.Ye, 43.35.Mr

INTRODUCTION

The determination of the viscosity of liquids is of great importance in investigating the internal structure of liquids as well as in controlling industrial processes employing oils, resins, or biological mixtures. Information about the viscosity of some organic liquids in the human body, such as blood or saliva, can be very useful in the medical diagnosis of certain diseases.

Using ultrasonic methods, one can determine the real R and imaginary X part of the shear impedance Z_l of the viscoelastic liquid. Ultrasonic methods are incomparable for frequencies above ~ 100 kHz. Shear bulk waves are highly damped in a viscoelastic liquid; i.e., their amplitude decreases e time after the fraction of the wavelength. Moreover, in the case of a Newtonian liquid, when $R \approx X$, the wavenumber of the *SH* (shear horizontal) bulk wave is pure imaginary. In this situation, the wave is then the nonpropagating mode. Therefore, the existing ultrasonic methods were reflectance methods, i.e., the shear impedance of the liquid was determined from phase $\Delta\theta$ and modulus r of the *SH* bulk wave reflected at the solid–liquid interface.[1] The considered methods were comparative; i.e., the results of two measurements were taken into account, the first for a free probe and the second for a probe loaded with the investigated liquid. Due to the big difference between the shear impedance of the liquid Z_l and the solid Z_q, the bulk wave method is of low sensitivity, e.g., for the probe of AT quartz $Z_q = 8.8 \times 10^6$ N s/m^3 loaded with H_2O one has $r = 0.997\,593$ and $\Delta\theta = 0.000\,241$ rad (Ref. 2), for frequency $f = 40$ MHz, and temperature $t_a = 25$ °C. These quantities are equivalent, respectively, to 0.0208 dB and 0.138° for one reflection of the *SH* bulk wave and, of course, are too small to be measured. For the 50th echo of the wave, the amplitude and phase changes are equal to 1 dB and 6.9°. To obtain an error of measurements lower than 10%, the amplitude and phase should be measured with an accuracy of 0.1 dB and 0.69° (for 40 MHz). These are rather strong requirements for electronic equipment. Moreover, the parallel faces of the probe should be polished with optical accuracy. Concluding, one can say that the classical bulk wave method is of low sensitivity *per se*. Therefore, a very precise electronic unit is required.

On the other hand, surface acoustic waves (SAW) propagating in solids are strongly dependent on the boundary condition on the surface of propagation: in particular, on the properties of an adjacent medium—liquid or gas, for instance.

SAW can be classified into two general groups: Rayleigh type waves and *SH* waves. The former waves have at least two components of vibration, i.e., longitudinal (L) and vertical transverse (*SV*), which cannot be separated. By contrast, *SH* surface waves possess only one *SH* component of vibrations. Therefore, they can be used to determine the shear parameters of an adjacent fluid. There are two well-defined types of *SH* surface waves; Love waves[3] and Bleustein–Gulyaev[4] waves. The former can propagate in layered subsurface structures and the latter can exist in some piezoelectric materials having at least a twofold axis of symmetry. Due to the characteristic dimension, i.e., thickness of the surface layer, surface waves of the Love type are always dispersive and exhibit a multimode structure. On the other hand, surface waves of the Bleustein–Gulyaev (B–G) type are nondispersive; this can be of some advantage during impulse measurements.

The other types of *SH* surface waves, such as surface skimming bulk waves (SSBW) or surface transverse waves (STW) are not taken into account in this paper.

Inspired by the facts presented above, the authors have proposed the application of *SH* surface waves propagating in solids to determine the rheological parameters of the adjacent viscoelastic liquid.[5]

It is interesting to note that surface waves of the Rayleigh type were employed to determine the acoustic impedance of nonviscous liquids for longitudinal waves.[6]

The fundamental principles of the proposed *SH* surface wave method are described in Sec. I. The appropriate analytical formulas are derived in Sec. II. Experimental procedures for measuring R and X are given in Sec. III. The parameters of the classical and the proposed methods are

 0001-4966/89/080818-10$00.80

Recent discoveries of the author in the field of Love waves

All these new discoveries were achieved using a full wave theory not a perturbation theory
Here, we can see the power of Mathematical Modeling

1. Analytical formulas for the mass sensitivity $S_{\sigma}^{v_p}$ of Love wave sensors

2. Proportionality between the mass sensitivity $S_{\sigma}^{v_p}$ and the relative slope of the dispersion curves $v_p(h)$ and $v_p(f)$

3. Love waves in lossy media

4. Counter-intuitive and unexpected phenomena in Love wave waveguides
   a) minimum of phase velocity as a function of liquid viscosity
   b) maximum of attenuation as a function of liquid viscosity

5. New mathematical tools applied in analysis of Love wave sensors.
   Inverse Problems

Mass sensitivity $S_{\sigma}^{v_p}$ of Love wave sensors

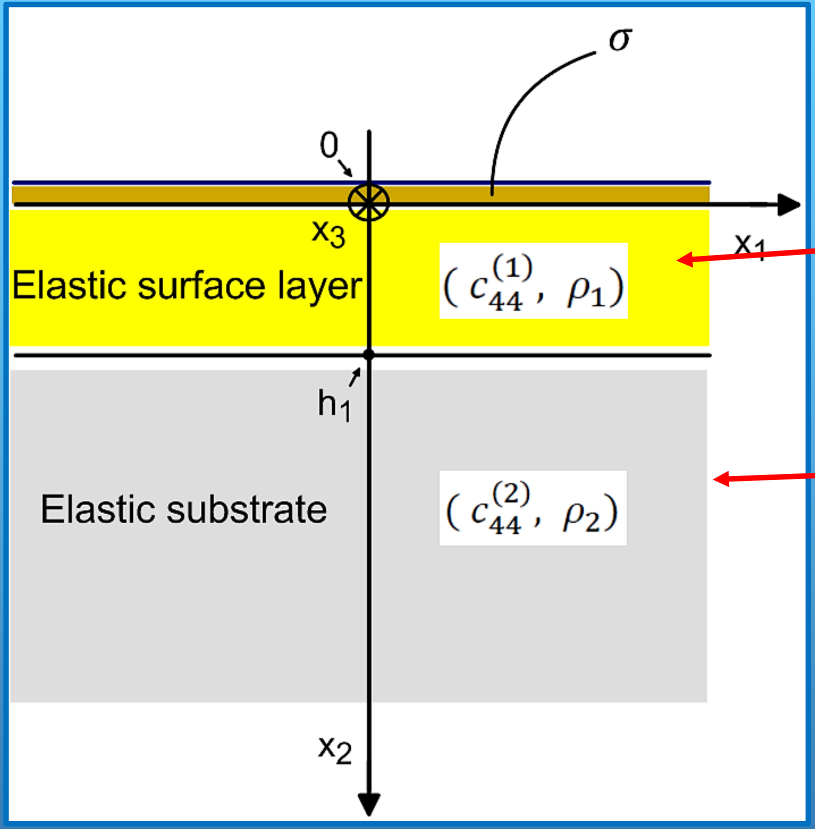


Fig.10. Physical model of the sensor.

Full-wave theory

$$\frac{1}{v_1^2}\frac{\partial^2 u_3}{\partial t^2} = \left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}\right)u_3$$

Equations of motion

$$\frac{1}{v_2^2}\frac{\partial^2 u_3}{\partial t^2} = \left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}\right)u_3$$

+ Boundary conditions

Dispersion equation:

$$tan(q_1 \cdot h_1)\cdot\left\{\left(c_{44}^{(1)}\cdot q_1\right)^2 + (\sigma\cdot\omega^2)\cdot\left(c_{44}^{(2)}\cdot b\right)\right\} + \left(c_{44}^{(1)}\cdot q_1\right)\cdot\left\{(\sigma\cdot\omega^2) - \left(c_{44}^{(2)}\cdot b\right)\right\} = F(v_p,\sigma,h_1,f) = 0 \qquad (20)$$

Mass sensitivity: $S_{\sigma}^{v_p} = \frac{1}{v_p}\cdot\frac{dv_p}{d\sigma}$

From the very sophisticated Implicit Function Theorem we get:

$$\frac{dv_p}{d\sigma} = -\frac{\partial F/\partial\sigma}{\partial F/\partial v_p} \qquad (19)$$

Here, we can see the power of Mathematics

σ is the surface mass density of an infinitesimally thin layer deposited on the waveguide surface

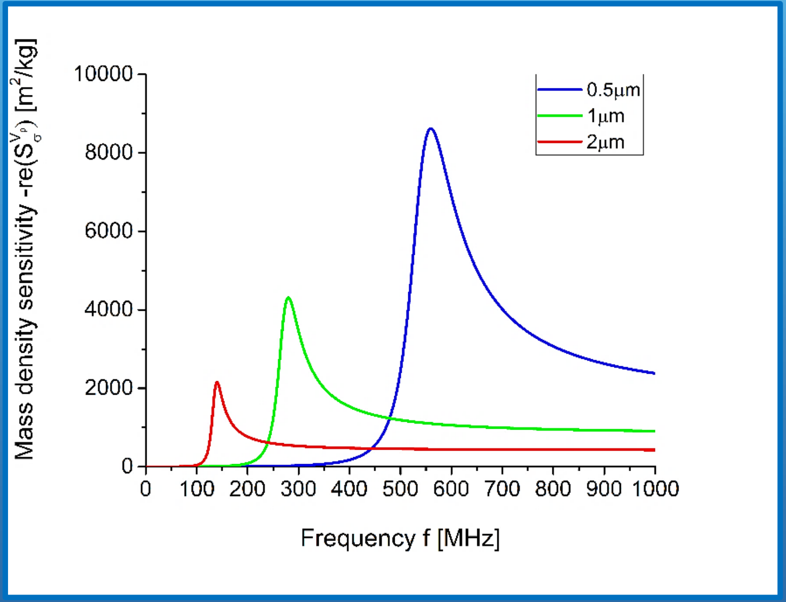
Full-wave mathematical model of the Love wave sensor.
P. Kiełczyński, Sensors & Actuators A, 2021, (to be published).

New original analytical formulas for the mass sensitivity $S_\sigma^{v_p}$ of Love wave sensors

$$S_\sigma^{v_p} = \frac{\omega^2 \frac{1}{k}\left\{\left(c_{44}^{(1)} q_1\right) + \left(c_{44}^{(2)} b\right)\cdot tan(q_1 h_1)\right\}}{\frac{h_1}{cos^2(q_1 h_1)}\frac{\partial q_1}{\partial k}\left\{\left(c_{44}^{(1)} q_1\right)^2 + \left(c_{44}^{(2)} b\right)(\sigma\omega^2)\right\} + tan(q_1 h_1)\left\{2q_1\left(c_{44}^{(1)}\right)^2\frac{\partial q_1}{\partial k} + c_{44}^{(2)}\frac{\partial b}{\partial k}(\sigma\omega^2)\right\} + c_{44}^{(1)}\frac{\partial q_1}{\partial k}\cdot\left\{(\sigma\omega^2) - \left(c_{44}^{(2)} b\right)\right\} - c_{44}^{(2)}\frac{\partial b}{\partial k}\left(c_{44}^{(1)} q_1\right)} \tag{21}$$

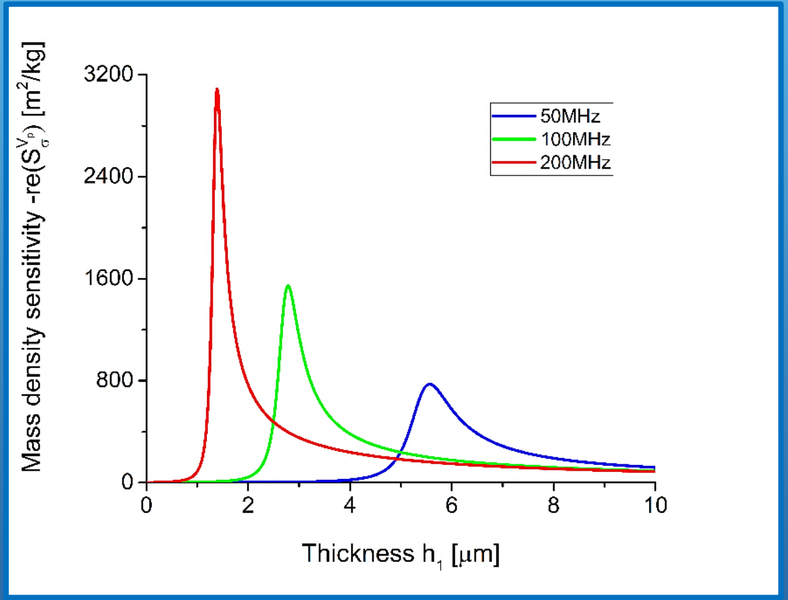
Huge advantage of analytical formulas: possibility of optimal selection of the thickess of the surface layer and material parameters of the waveguide

Fig.11.



Mass sensitivity $S_\sigma^{v_p}$ versus wave frequency

Fig.12.



Mass sensitivity $S_\sigma^{v_p}$ versus surface layer thicknees

Discovery of the proportionality between the mass sensitivity $S_\sigma^{v_p}$ and phase velocity gradients

(22) $$S_\sigma^{v_p} = \frac{\partial F/\partial \sigma}{\partial F/\partial h_1} \cdot \frac{1}{v_p} \cdot \left(\frac{dv_p}{dh_1}\right)$$

(23) $$S_\sigma^{v_p} = \frac{\omega^2 \cdot \cos^2(q_1 h_1)}{q_1} \cdot \frac{\left\{\tan(q_1 h_1) \cdot \left(c_{44}^{(2)} b\right) + \left(c_{44}^{(1)} q_1\right)\right\}}{\left\{\left(c_{44}^{(1)} q_1\right)^2 + (\sigma\omega^2) \cdot \left(c_{44}^{(2)} b\right)\right\}} \cdot \frac{1}{v_p} \cdot \left(\frac{dv_p}{dh_1}\right)$$

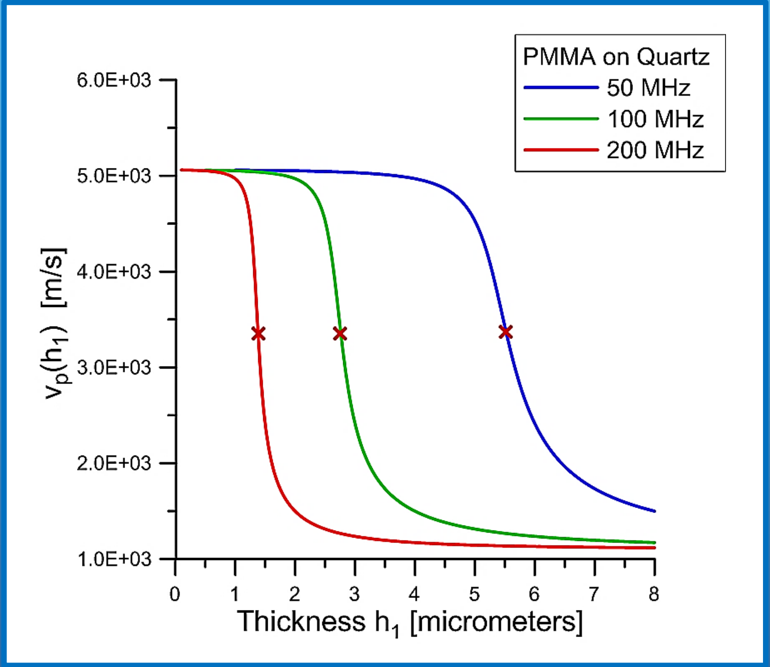


Fig.13.

The highest sensitivity $S_\sigma^{v_p}$ occurs at the points where the dispersion curves are the steepest

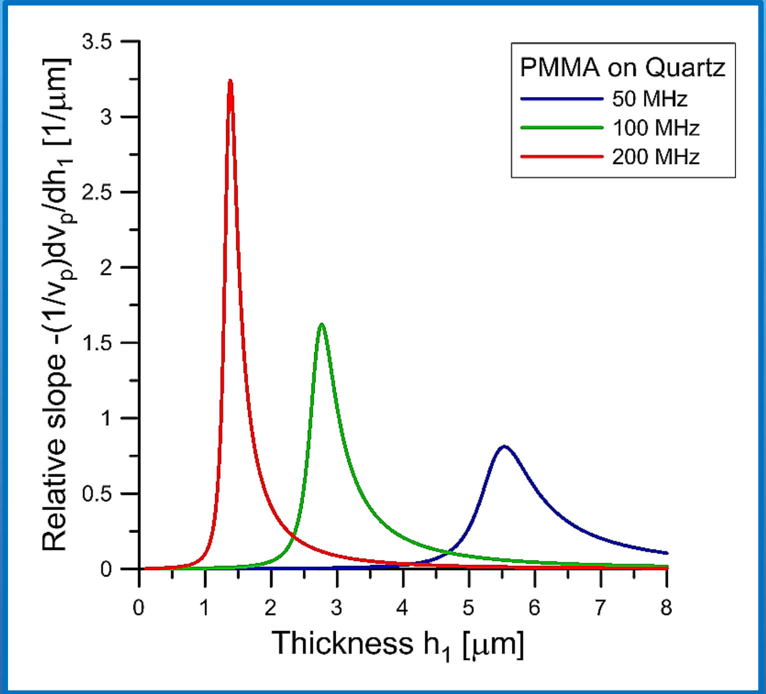


Fig.14.

Phase velocity $v_p(h_1)$ versus surface layer thicknees.

Relative slope $\frac{1}{v_p} \cdot \left(\frac{dv_p}{dh_1}\right)$ versus surface layer thicknees

Perturbative formula for $S_\sigma^{v_p}$: Mc Hale et al., Journal of Applied Physics, 2002

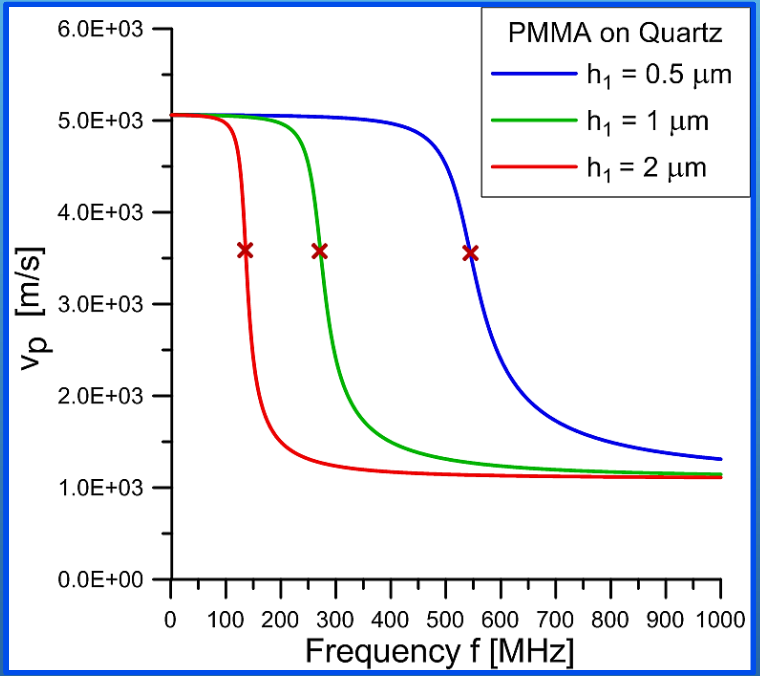
Exact analytical formulas for the mass sensitivity $S_\sigma^{v_p}$ in terms of the phase velocity gradients

(24) $$S_\sigma^{v_p} = \frac{\partial F/\partial\sigma}{\partial F/\partial f}\cdot\frac{1}{v_p}\left(\frac{dv_p}{df}\right)$$

$$S_\sigma^{v_p} = \frac{\omega^3}{2\pi}\cdot\frac{\tan(q_1h_1)\cdot\left(c_{44}^{(2)}b\right)+c_{44}^{(1)}q_1}{\frac{q_1h_1}{\cos^2(q_1h_1)}\left[\left(c_{44}^{(1)}q_1\right)^2+\sigma\omega^2c_{44}^{(2)}b\right]+\tan(q_1h_1)\cdot\sigma\omega^2c_{44}^{(2)}b+\sigma\omega^2c_{44}^{(1)}q_1}\cdot\frac{1}{v_p}\cdot\left(\frac{dv_p}{df}\right)$$ (25)

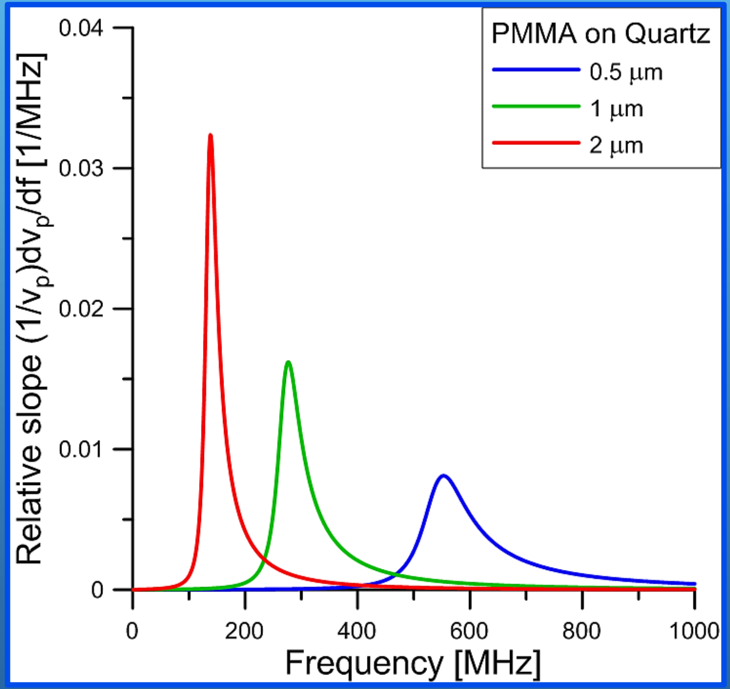
Exact (closed-form) analytical formula

Fig.15.



The highest sensitivity $S_\sigma^{v_p}$ occurs at the points where the dispersion curves are the steepest

Fig.16.



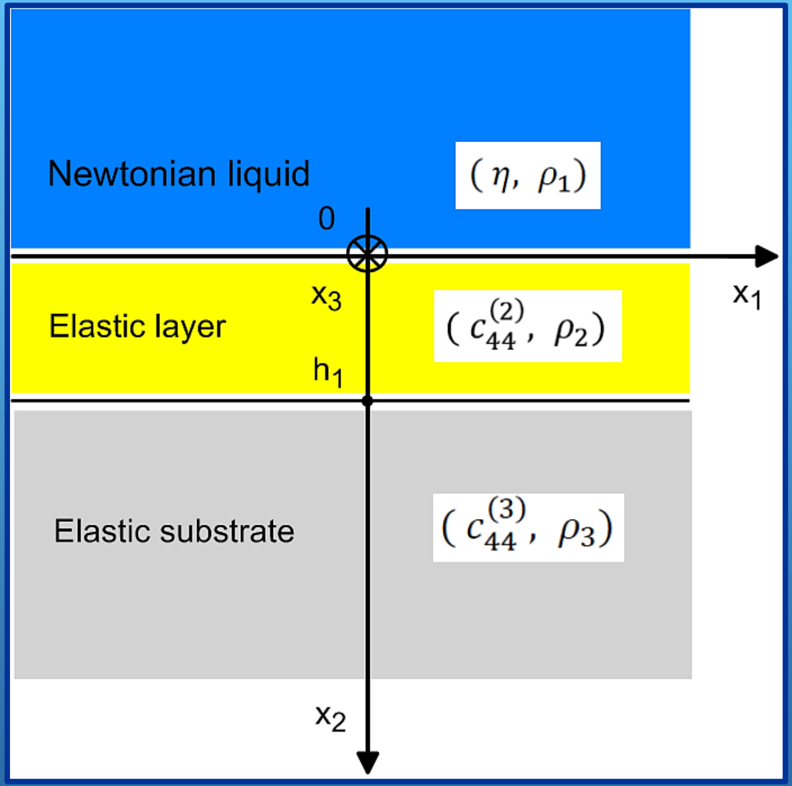
Plot of the phase velocity versus frequency $v_p = v_p(f)$

Plot of the relative slope versus frequency $S_\sigma^{v_p} = S_\sigma^{v_p}(f)$

New unexpected counter intuitive phenomena displayed by Love waves in waveguides loaded with a lossy liquid

Love waves in lossy waveguides reveal a number of new amazing propertiers.
Let the surface of the waveguide is covered with a lossy liquid (e.g., Newtonian one)

Fig.17.



Equations of motion

Full-wave theory

Complex Dispersion Equation (26)

$$\sin(qD)\cdot\{(\mu_1)^2\cdot q^2+\mu_2\cdot b\cdot\lambda_1\cdot j\omega\eta\}-\cos(qD)\cdot\{\mu_1\cdot\mu_2\cdot b\cdot q-\mu_1\cdot q\cdot\lambda_1\cdot j\omega\eta\}= \\ =0$$

Quantities in red q, b *and* λ_1 in Eq.26 are complex: $\beta=\beta_0+j\alpha$

(P. Kiełczyński et al., „Effect of a viscous liquid loading on Love wave propagation", International Journal of Solids and Structures, 2012)

Love wave waveguide is covered with a lossy liquid

Propagation constant and transverse wave numbers of the Love wave are complex

Minimum of phase velocity and maximum of attenuation as a function of liquid viscosity (waveguide Fig.17)

Using a full-wave theory we received the following unexpected results:

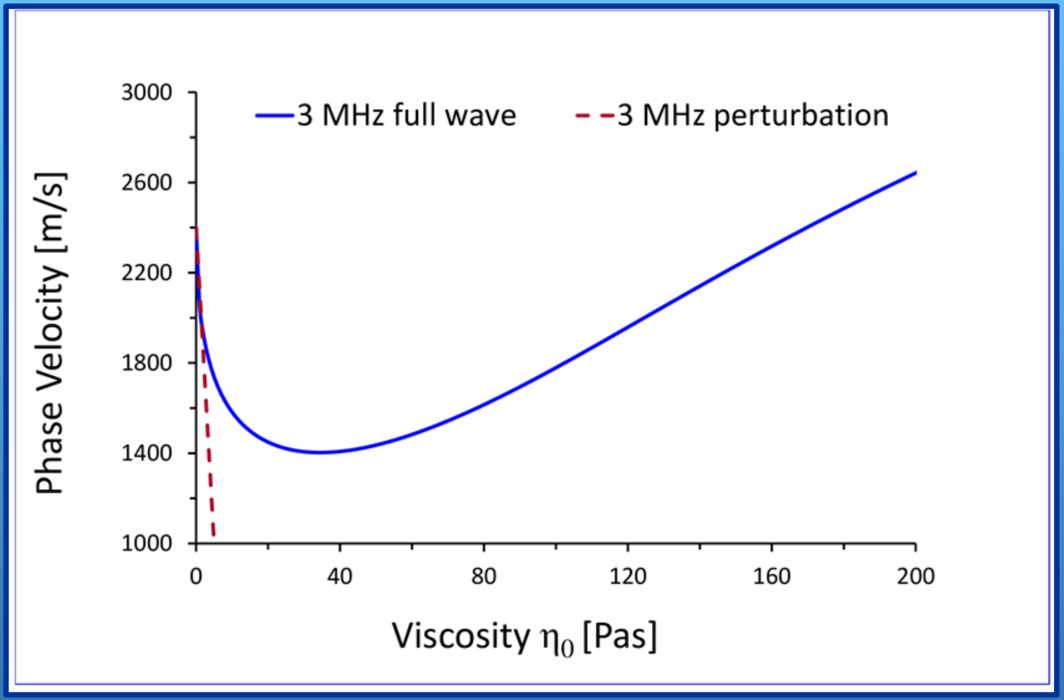


Fig. 18. Phase velocity as a function of liquid viscosity

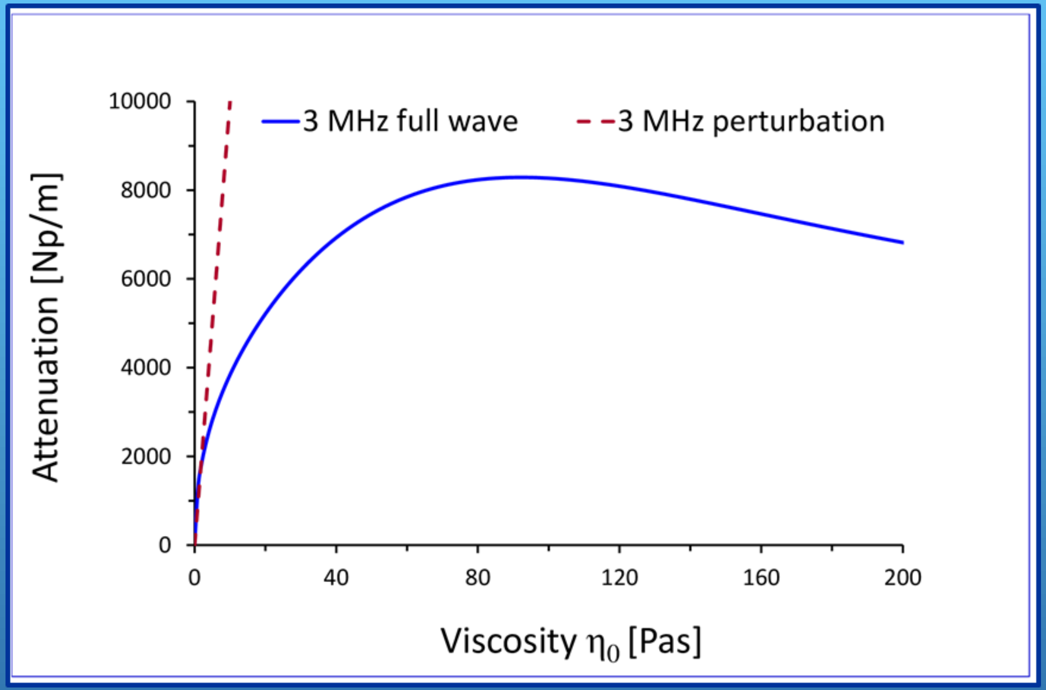


Fig. 19. Attenuation versus liquid viscosity

Applying a full-wave theory, we got unexpected results that are not provided by the perturbation theory (straight red lines)

Sudden qualitative changes in phase velocity and attenuation of the Love wave (continuation)

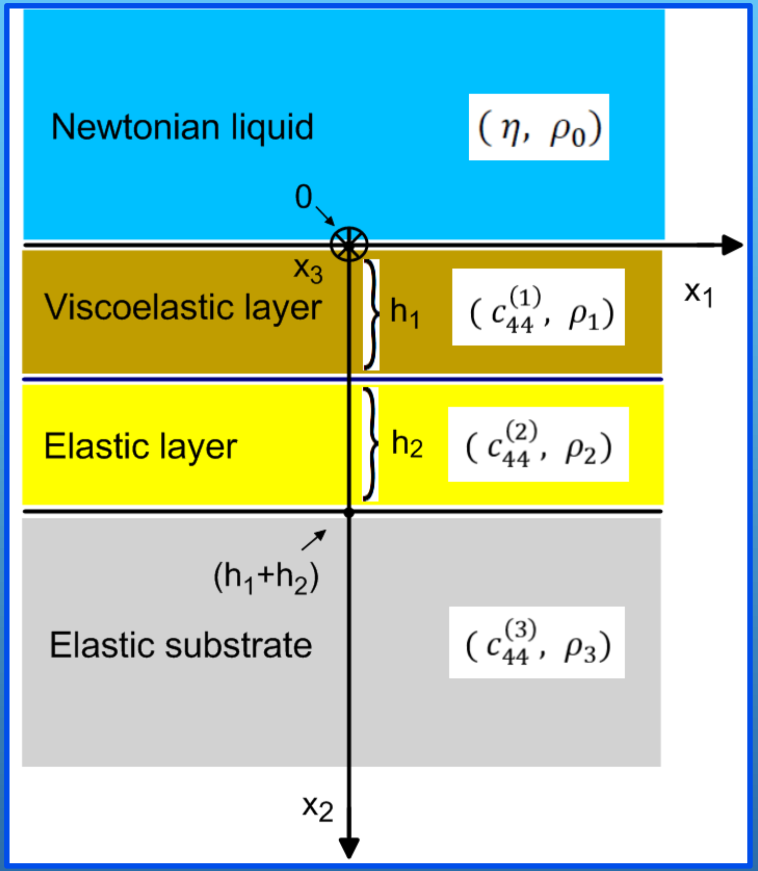


Fig.20. Cross-section of the analyzed 2 surface layer Love wave waveguide.

Novel dispersion equation for 2 surface layer waveguides loaded with a Newtonian liquid:

$$-\tan(q_1 \cdot h_1) \cdot \tan(q_2 \cdot h_2) \cdot \left\{ \frac{\left(\lambda_1 \cdot c_{44}^{(0)}\right)}{\left(c_{44}^{(3)} \cdot b\right)} \frac{\left(c_{44}^{(2)} \cdot q_2\right)}{\left(c_{44}^{(1)} \cdot q_1\right)} + \frac{\left(c_{44}^{(1)} \cdot q_1\right)}{\left(c_{44}^{(2)} \cdot q_2\right)} \right\} + \\ + \tan(q_1 \cdot h_1) \cdot \left\{ \frac{\left(\lambda_1 \cdot c_{44}^{(0)}\right)}{\left(c_{44}^{(1)} \cdot q_1\right)} - \frac{\left(c_{44}^{(1)} \cdot q_1\right)}{\left(c_{44}^{(3)} \cdot b\right)} \right\} + \\ + \tan(q_2 \cdot h_2) \cdot \left\{ \frac{\left(\lambda_1 \cdot c_{44}^{(0)}\right)}{\left(c_{44}^{(2)} \cdot q_2\right)} - \frac{\left(c_{44}^{(2)} \cdot q_2\right)}{\left(c_{44}^{(3)} \cdot b\right)} \right\} + \left(\frac{\left(\lambda_1 \cdot c_{44}^{(0)}\right)}{\left(c_{44}^{(3)} \cdot b\right)} + 1 \right) = 0 \qquad (27)$$

Full-wave theory

Another unexpected result: sudden qualitative changes in phase velocity and attenuation of the Love wave

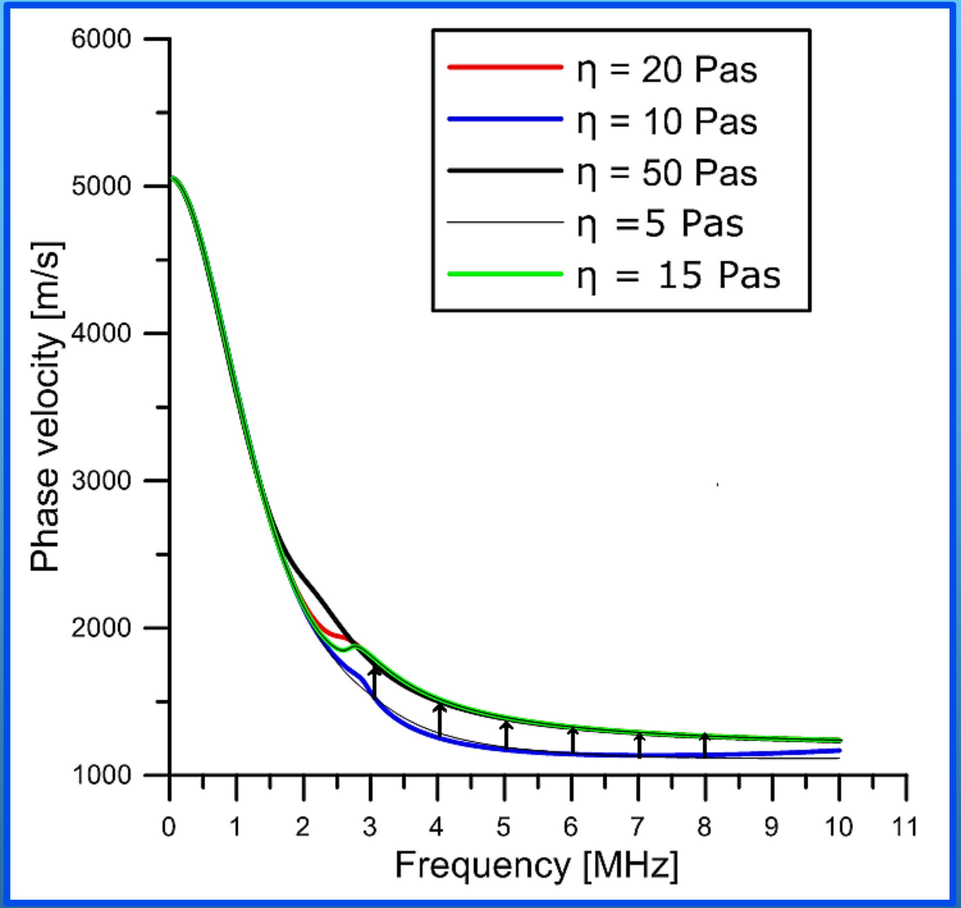


Fig.21. Phase velocity versus frequency

Viscosity of the loading viscoelastic liquid increases gradually from η_0=0 Pas.
For a certain value of the viscosity η_0~ ~ 11 Pas, we observe an unexpected and dramatic change in the phase velocity and attenuation of the Love wave.

$\eta = \eta_0 \sim 11\ Pas$

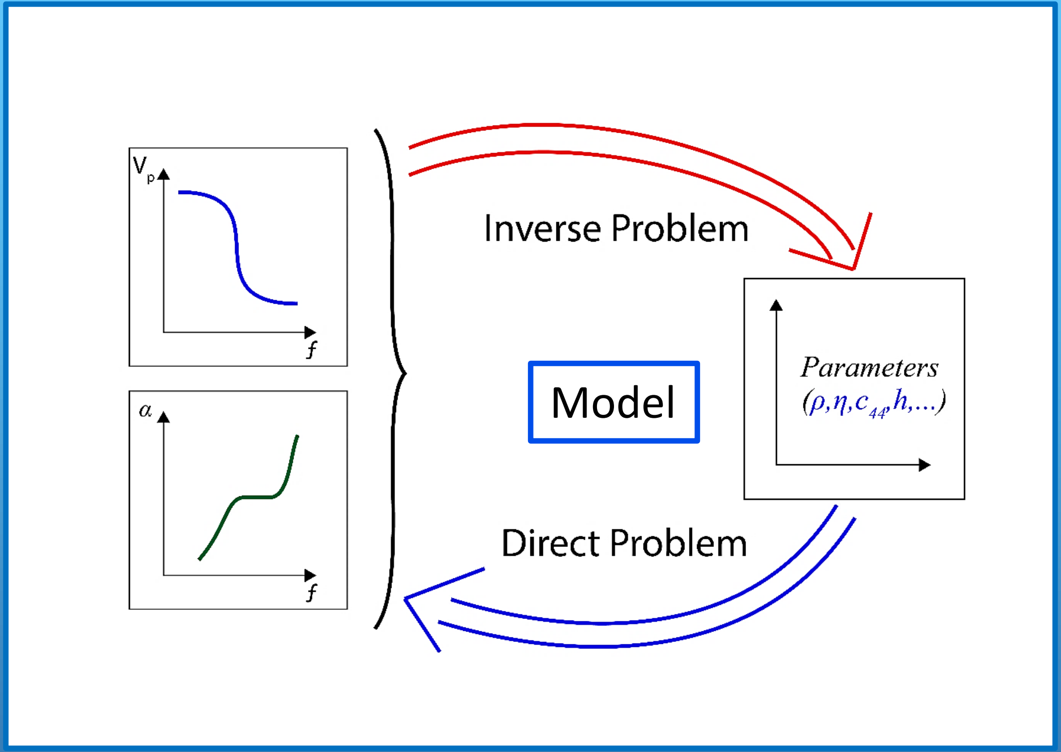
η

$\eta = 0\ Pas$

Elastic waveguide has 2 different surface layers (Fig.21)
1st elastic surface layer = PMMA
2nd elastic surface layer = Gold
Substrate = Quartz

No such effect is observed in a single layer waveguide

New mathematical tools applied by the author in analysis of Love wave sensors



Direct Sturm-Liouville Problem

Fig.22.

Inverse Sturm-Liouville Problem

The solution of the Inverse Problem is equivalent to the solution of the Optimization Problem:
min Φ(analyte, waveguide, experiment, ω; X) ; $\Phi(X) \geq 0$; Φ is an Objective Function
seeking for X that minimizes the Objective Function Φ ; X material parameters

Application of Inverse Problems to simultaneous determination of density and viscosity (ρ, η) of liquids using Love surface waves

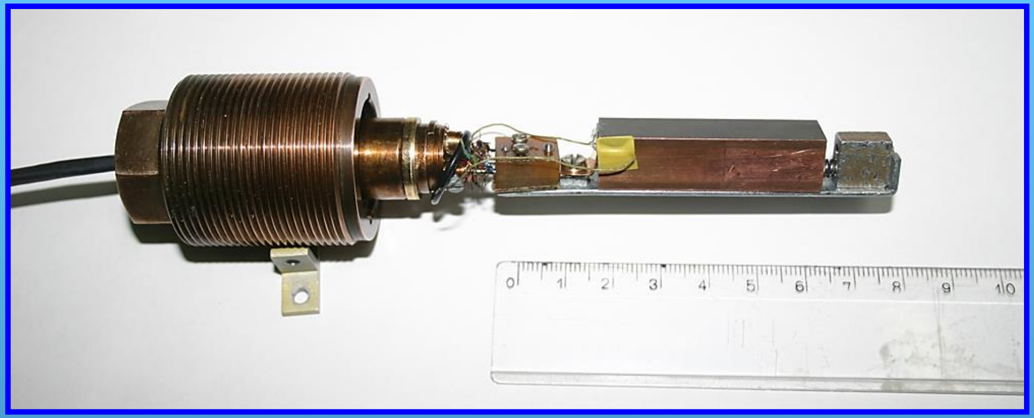


Fig.23.

Sensor = only a single Love wave waveguide !!!
No lasers, no magnetic fields

I have achieved excellent results:

1) P. Kiełczyński et al., „Inverse procedure for simultaneous evaluation of viscosity and density of Newtonian liquids from dispersion curves of Love waves", Journal of Applied Physics, 116, (2014) 044902

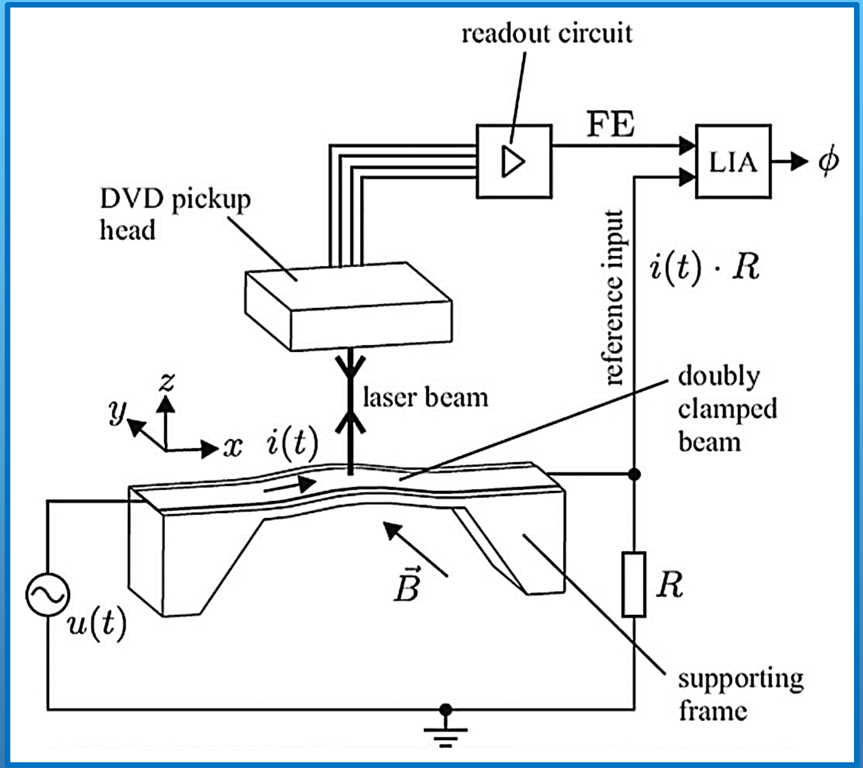


Fig.24.

1) Jakoby et al., IEEE, Trans on UFFC, 2010
cantilevers + magnetic field = (quite a complicated)

2) Herrmann et al., , Applied Physics Letters, 1999

Future Direction of Research and Perspectives for Love Wave Sensors

New Analytical Methods:

1) Inverse problems (higher accuracy),

   Minimization of the Objective Function Φ:

   min Φ (analyte, waveguides, experiment, ω)

2) Optimization methods in Banach space (Functional Analysis)

New materials and waveguide structures:

1) Multilayer (N>10) LSW waveguides (wideband characteristics)
2) Higher operating frequencies 2-5 GHz (higher sensitivity)
3) New fast materials for the substrate: Diamond, (BN) boron nitride, (AlN) aluminum nitride